

# An Exploration of Learners' Understanding of Euclidean Geometric Concepts: A Case Study of Secondary Schools in the OR Tambo Inland District of the Eastern Cape



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## ABSTRACT

A major concern in South Africa is the poor performance of learners in mathematics, particularly in geometry. This paper therefore sought to explore the learners' understanding of Euclidean geometric concepts. The Van Hiele model was a useful framework for understanding the reasoning and challenges that students encounter with geometry. This study focused on 15 participants from rural South African schools, including 5 grade 10 mathematics learners in each of the three secondary schools and three teachers who are teaching mathematics in grade 10, in three secondary schools. Through face-to-face interviews, learners' comprehension of geometry was qualitatively assessed. Pedagogical and methodological difficulties, lack of learners' interest and comprehension of numerous geometrical concepts, as well as the absence of technology use, were found to contribute to the challenges in learning and teaching Euclidean geometry. The recommendations suggest that teachers should plan and prepare their geometry classes with the students' understanding in mind that Euclidean geometry has been a cornerstone of mathematical education for centuries, teaching students critical thinking, problem-solving, and logical reasoning skills. This study adds to the existing literature on introducing new concepts in mathematics into the educational system of South Africa.

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## INTRODUCTION

The aim of this paper is to investigate how well learners comprehend Euclidean geometric concepts. Geometry, particularly Euclidean geometry, is a crucial component of the mathematics curriculum that creates a strong basis for success in science, engineering, and technology.<sup>1</sup> The performance of learners in geometry has declined, and many believe that it has no practical application. However, studying geometry can help develop important skills like visualization, critical thinking, problem-solving, conjecturing, deductive reasoning, logical argument, and proof which are essential for success in today's

<sup>1</sup> Heri Retnawati, Janu Arlinwibowo, and Eny Sulistyanyingsih, "The Students' Difficulties in Completing Geometry Items of National Examination," *International Journal* 8, no. 4 (2017): 03.

world. Despite its challenges, geometry is still a valuable subject to learn.<sup>2</sup> Teaching and learning Euclidean geometry can be difficult for both students and teachers. Common challenges include disinterest from students, difficulty understanding certain concepts, limited access to technology, inadequate foundational knowledge, lack of motivation, insufficient teacher training, and an imbalanced teacher-student ratio.<sup>3</sup> To improve learner performance, it is necessary to identify the specific geometry topics that students struggle with and support teachers who lack confidence in teaching the subject.<sup>4</sup> It is important to address certain issues that students may face during exams, such as the lack of physical models, difficulty solving geometry problems, and understanding three-dimensional shapes. Students may struggle with both procedural and conceptual understanding, recognizing relationships between properties and shapes, and comprehending geometric language.<sup>5</sup>

Despite its difficulties, geometry is still a significant component of the final National Curriculum Statement Grade 12 paper in South Africa, comprising around 30% of the subject.<sup>6</sup> The blended learning model combines online learning with traditional classroom teaching.<sup>7</sup> Teachers can use a combination of in-person and online instruction to enhance students' understanding of a course. This approach is based on how students perceive the world and their development of geometric thinking, according to Van Hiele's theory. While there is some support for the hypothesis of Dutch mathematics teachers Pierre Van Hiele and Diana Van Hiele-Geldof, others disagree. Experts in the field are working to determine the stages of geometric knowledge that children go through from an early age until they acquire correct abstract levels and axiomatic geometric ideas. The van Hiele model of geometric cognition describes how children develop spatial geometry concepts, suggesting five levels of geometric thinking that students can progress through to develop their geometric reasoning skills. These levels include visual, analysis, informal deduction, formal deduction, and rigor, with specific words and phrases used to describe the changes in one's thought process at each level. Ismail and Rahman suggest that teachers are responsible for selecting suitable learning experiences for students in different classroom settings and at various levels.<sup>8</sup> Visual learners have a straightforward understanding of space and perceive geometric figures or shapes as complete units. They differentiate between geometric shapes based on appearance rather than qualities. Learners can recognize shapes provided to them by relating them to their existing knowledge. However, they cannot evaluate the shapes' characteristics without observation.<sup>9</sup> Students who have achieved analysis-level proficiency can break down shapes into their individual components and properties, but they may struggle to establish connections between different shapes.<sup>10</sup>

As a student at the analysis level, one should be able to recognize a square as a shape with four equal sides and angles. This knowledge enables them to logically categorize it. Asemani, Asiedu-Addo, and Oppong suggest that as students advance to the informal deduction level, they can assess the features of shapes and understand the links between them.<sup>11</sup> As students progress to the formal deduction level, they gain the ability to comprehend and apply formal geometry concepts. They can establish geometric theories based on axioms and arguments, like Euclidean geometry. Modern students are taught formal proofs that follow logical arguments based on shape attributes, but they may not be able to create original proofs. At the highest level of Van Hiele's geometric learning theory, known as axiomatization, students

<sup>2</sup> Robin B. Dimla, "Probing Students' Levels of Geometric Thinking in Geometry and Their Enacted Example Space Function," *Journal of Education in Black Sea Region* 4, no. 1 (2018): 155–63.

<sup>3</sup> D. Rohendi, S. Septian, and H. Sutarno, "The Use of Geometry Learning Media Based on Augmented Reality for Junior High School Students," in *IOP Conference Series: Materials Science and Engineering*, vol. 306 (IOP Publishing, 2018), 012029.

<sup>4</sup> A. Giannakopoulos, "An Alternative Way of Solving Geometry Riders in Grade 12: Back to Synthesis and Analysis," *Proceedings of the 23rd Annual National Congress of the Association for Mathematics Education of South Africa* (Johannesburg: AMESA, 2017).

<sup>5</sup> Acep Rohendi, "Consumer Protection in the E-Commerce: Indonesian Law and International Law Perspective," *Berkman Center Research Publication*, Forthcoming, 2015.

<sup>6</sup> Department of Basic Education (DBE). *Curriculum and Assessment Policy Statement. Mathematics Grades 10-12*. (Pretoria: DBE, 2011).

<sup>7</sup> D. L. Sulistiowati, T. Herman, and A. Jupri, "Student Difficulties in Solving Geometry Problem Based on Van Hiele Thinking Level," *Journal of Physics: Conference Series* 1157 (February 2019): 042118, <https://doi.org/10.1088/1742-6596/1157/4/042118>.

<sup>8</sup> Lilliati Ismail et al., "The Effectiveness of Using a Tracker Chart to Enhance Willingness to Communicate among ESL Learners in a Philosophical Inquiry Classroom Discussion.," *Pertanika Journal of Social Sciences & Humanities* 28 (2020).

<sup>9</sup> William F. Burger and J. Michael Shaughnessy, "Characterizing the Van Hiele Levels of Development in Geometry," *Journal for Research in Mathematics Education* 17, no. 1 (January 1986): 31–48, <https://doi.org/10.5951/jresmetheduc.17.1.0031>.

<sup>10</sup> John Mason, "Attention and Intention in Learning about Teaching through Teaching," in *Learning through Teaching Mathematics: Development of Teachers' Knowledge and Expertise in Practice* (Springer, 2010), 23–47.

<sup>11</sup> Emmanuel Asemani, Samuel K. Asiedu-Addo, and Richard A. Oppong, "The Geometric Augmented Reality for Junior High School Students," in *IOP Conference Series: Materials Science and Engineering*, 1st ed., vol. 306 (IOP Publishing, 2018), 012029.

can compare and reason about a range of mathematical systems, including Euclidean and non-Euclidean geometry, without relying on physical models. The researcher believes that altering how geometry is taught in schools can assist children in accepting different geometrical representations and thinking abstractly about systems. This study thus seeks to investigate how students perceive geometry using Van Hiele's degrees of geometric thinking, with a specific focus on grade 10 of the Further Education and Training (FET) academic study range.

## LITERATURE REVIEW

The ensuing literature review delves into the intricacies of teaching Geometry, highlighting the challenges that often arise in the process of learning Euclidean Geometry. It also explores the various factors that impact a learner's geometric performance and evaluates the strategies that can be employed to enhance the teaching and learning of Geometry.

### Teaching Geometry

A basic understanding of space and shapes is a prerequisite for learning geometry. Different placements of shapes and the capacity to conceptualize and express the movement of objects using words are examples of spatial thinking.<sup>12</sup> This includes for example backwards, forwards, left and right, up and down. Shape refers to both two-dimensional and three-dimensional shapes and their form. For instance, moving up, down, left, and right are examples of this. The average pass rate for Grade 12 mathematics learners in South Africa is less than 60%, according to a review of the diagnostic analysis for the Grade 12 National Senior Certificate test from 2016 to 2020.<sup>13</sup> The claim is supported by an examination of the National Senior Certificate (NSC) diagnostic report for each question, which reveals a decline in student performance in Mathematics Paper 2, particularly in Euclidean geometry.<sup>14</sup> To cope with the problem, Machisi suggests using unconventional teaching methods, such as the Van Hiele theory-based teaching and learning technique, which, in his words, "meets learners' needs better than conventional approaches in learning Euclidean geometry." Sfard also recommended that teachers attempt to increase their students' discourse involvement in order to lessen the difficulties that mathematics learners face.<sup>15</sup> Since the subject involves practical actions, teachers lack both subject-specific knowledge and an understanding of alternative teaching methods. The importance of geometrical abilities is equally crucial in the building industry. Geometrical proficiency is also essential for engineering, architectural design, and construction work.<sup>16</sup> Teachers migrate from a conceptual approach (the accurate application of procedures) to a procedural approach (exact computation) in many geometry classes across the country through immersive and flexible instruction and learning. Many academics, like Brown, claim that students' understanding of geometrical ideas, logic, and problem-solving skills is insufficient. In most geometry classes across the nation, teachers transition from a conceptual approach (the accurate application of procedures) to a procedural approach (calculation accuracy) through immersive and adaptable education and learning. According to several academics, like Brown, pupils' comprehension of geometrical concepts, reasoning, and problem-solving abilities are lacking.<sup>17</sup>

<sup>12</sup> P. Olley et al., "Analysis and Design of a Centrifugal Filter Geometry for Entrained Particle Separation," *Journal of Aerosol Science* 41, no. 9 (2010): 848–58, <https://doi.org/10.1016/j.jaerosci.2010.05.004>.

<sup>13</sup> Department of Education. "A review of the Grade 12 National Senior Certificate" 2016, 2017, 2018, 2019.

<sup>14</sup> Thabani Chihambakwe, "Impact of National Senior Certificate Examination Diagnostic Report on Learner Performance in Mathematics" (University of Zululand, 2017).

<sup>15</sup> Anna Sfard, *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing* (Cambridge university Press, 2008).

<sup>16</sup> Sarah Bansilal and Ifunanya Ubah, "The Use of Semiotic Representations in Reasoning about Similar Triangles in Euclidean Geometry," *Pythagoras* 40, no.1(2019): 1–10; J. K. Alex and K. J. Mammen, "A Survey of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the Van Hiele Theory," *The Anthropologist* 14, no. 2 (2012): 123–29.

<sup>17</sup> Gladys Sunzuma and Aneshkumar Maharaj, "In-Service Teachers' Geometry Content Knowledge: Implications for How Geometry Is Taught in Teacher Training Institutions," *International Electronic Journal of Mathematics Education* 15, no. 1 (2019).

## Challenges of Teaching and Learning Euclidean Geometry

One of the most challenging academic subjects to learn and master is mathematics.<sup>18</sup> Despite the teachers' best attempts to explain how to approach the exercises because mathematics is seen as a hard topic, the learner's achievements were only average.<sup>19</sup> The educational system, however, aims to encourage math teaching and proficiency.<sup>20</sup> Mathematics is an activity subject.<sup>21</sup> Therefore, the method of instruction is vital in assisting students in developing core mathematical knowledge, talents, and attitudes that will improve their memory of mathematical concepts so they may find solutions to a variety of issues in their daily lives.<sup>22</sup> Geometry is an essential part of the school curriculum globally from grade R to grade 12.<sup>23</sup> Schools adopt strengthening curricula in several nations, including South Africa, where subjects like geometry are incrementally built upon each year.<sup>24</sup> The foundation phase of the South African curriculum begins with the introduction of fundamental two-dimensional and three-dimensional shapes before moving on to the study of Euclidean geometry.

## Factors Affecting Learners' Geometrical Performance

One of the significant factors contributing to the student's difficulty learning mathematics is the phenomenon known as "mathematics anxiety."<sup>25</sup> According to Estonanto and Dio, participants' exposure to abstract mathematical concepts, the teacher's teaching style and attitude, and their lack of analytical and comprehension skills were the main contributors to their mathematics anxiety. The phenomenon known as "mathematics anxiety" is one of the important aspects leading to the pupil's difficulty in learning mathematics. Estonanto and Dio stated that the leading causes of the participants' mathematics anxiety included their exposure to abstract mathematical concepts, the teacher's teaching style and attitude, and their lack of analytical and understanding skills.<sup>26</sup> According to studies, many teachers struggle with math performance and anxiety, particularly elementary school teachers who frequently lack familiarity with mathematical ideas and experience higher levels of arithmetic anxiety than the average college learner.<sup>27</sup> The main challenge found by this study was that the participants lacked a conceptual understanding of geometric concepts. Nogues and Dorneles identified that learners' achievement of mathematical knowledge largely depends on cognitive competence.<sup>28</sup> Euclidean geometry hones essential visual, logical, rational, and problem-solving skills. Nevertheless, despite numerous justifications for including Euclidean geometry in secondary school mathematics curricula, the teaching of this mathematical topic has been marked by significant pedagogical difficulties in many nations, including South Africa,<sup>29</sup> Malawi, Namibia, Nigeria, Zimbabwe, Ghana, America, Saudi Arabia, and Turkey.

<sup>18</sup> Cristobal A. Rabuya, "Factors Related to Mathematics Anxiety among Senior High School Students in Basic and Pre-Calculus: A Descriptive CROSS-Sectional Psychological Study," *Journal for ReAttach Therapy and Developmental Diversities* 6, no. 9s(2023):183–90.

<sup>19</sup> Nelia M. Adora, "Mathematics Performance of Grade Six Pupils in the National Achievement Test, Division of Northern Samar..." in *Universal Academic Cluster International May Conference in Bangkok*, 2017, 25–26.

<sup>20</sup> Pilar Ester et al., "The Verbal Component of Mathematical Problem Solving in Bilingual Contexts by Early Elementary Schoolers," *Mathematics* 9, no. 5 (March 6, 2021): 564, <https://doi.org/10.3390/math9050564>.

<sup>21</sup> Grace N. Okoye and Edith C. Onyeka, "Application of Geotrigonometric Set and Geoboard on Students' retention in Mathematics in Rivers State Nigeria," *African Journal of Educational Management, Teaching and Entrepreneurship Studies* 7, no. 2 (2022): 82–97.

<sup>22</sup> Edith C. Onyeka and G.N.Okoye, "Effectiveness of Demonstration Teaching Method on Student Academic Achievement in Mathematics in Rivers State Nigeria," *African Journal of Educational Management, Teaching and Entrepreneurship Studies* 8, no. 2 (2023): 32–42.

<sup>23</sup> Jogymol K. Alex and Kuttickattu J. Mammen, "Lessons Learnt From Employing van Hiele Theory Based Instruction In Senior Secondary School Geometry Classrooms," *EURASIA Journal of Mathematics, Science and Technology Education* 12, no.8(July 2, 2016).

<sup>24</sup> Department of Basic Education, *Curriculum News: Improving the Quality of Learning and Teaching-Strengthening Curriculum Implementation from 2010 and Beyond* (Pretoria: Government Printers, 2011).

<sup>25</sup> Anthony Siaw et al., "The Impact of Internet Use on Income: The Case of Rural Ghana," *Sustainability* 12, no. 8 (April 17, 2020): 3255.

<sup>26</sup> Aldrin J.Estonanto and Ryan V Dio, "Factors Causing Mathematics Anxiety of Senior High School Students in Calculus," *Asian Journal of Education and E-Learning (ISSN: 2321-2454)* 7, no. 01 (2019).

<sup>27</sup> Elena Novak and Janet L.Tassell, "Studying Preservice Teacher Math Anxiety and Mathematics Performance in Geometry, Word, and Non-Word Problem Solving," *Learning and Individual Differences* 54 (February 2017): 20–29.

<sup>28</sup> Camila P. Nogues and Beatriz V. Dorneles, "Systematic Review on the Precursors of Initial Mathematical Performance," *International Journal of Educational Research Open* 2 (2021): 100035, <https://doi.org/10.1016/j.ijedro.2021.100035>.

<sup>29</sup> Jayaluxmi Naidoo and Winifilda Kapofu, "Exploring Female Learners' Perceptions of Learning Geometry in Mathematics," *South African Journal of Education* 40, no. 1 (2020): 1–11; Harrison Ngirishi and Sarah Bansilal, "An Exploration of High School Learners' Understanding of Geometric Concepts," *Problems of Education in the 21st Century* 77, no. 1 (2019): 82; Simon A. Tachie, "The Challenges of South African Teachers in Teaching Euclidean Geometry," *International Journal of Learning, Teaching and Educational Research* 19, no. 8 (2020): 297–312; Lisnet Mwadzaangati, "Mathematical Knowledge for Teaching Geometric Proof: Learning from Teachers' Practices," *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education*, 2015; Leena N Kanandjebo and Elizabeth N. Ngololo, "The Effects of 'Geometry Sketchpad' on Grade 12 Learners' Performance in Geometry," 2017;

## Strategies to enhance teaching and learning of Geometry

Many students find it difficult to learn geometry because they lack the conceptual understanding and geometrical problem-solving skills required. According to Van Hiele, this is because historically, teachers have taught at a level above that of the students.<sup>30</sup> According to Alex and Mammen, teachers' failure to implement efficient and pertinent teaching strategies that would actively engage students and efficiently utilize their cognitive capacities may be a contributing factor to students' lack of geometric thinking.<sup>31</sup> According to Shulman, a teacher needs to have three different categories of knowledge in order to successfully instruct students.<sup>32</sup> They include curriculum knowledge, pedagogical content knowledge, and content knowledge. However, pedagogical content knowledge is prioritized highly because it identifies the distinctive bodies of knowledge for teaching. Rahmiati and Scandrett suggest the use of instructional materials as one of the resolutions to promote active learning in the classroom in order to improve learners' performance in geometric concepts.<sup>33</sup> Instructional materials are educational tools used to improve learners' learning skills and understanding, and also to increase learners' motivation levels of geometric understanding.<sup>34</sup> In order to increase learners' performance with geometric concepts, Rahmiati and Scandrett propose the usage of instructional resources as one of the resolutions to encourage active learning in the classroom.<sup>35</sup> According to Sibiya, instructional materials are educational resources that are utilized to enhance students' comprehension and learning abilities as well as their motivation to comprehend geometry.<sup>36</sup> Additionally, these resources encourage active participation from students, fostering an environment conducive to learning.<sup>37</sup> Jones concurs that students become very engaged in activities where they may create mathematical definitions and identify mathematical qualities when they use educational tools like GeoGebra and Geoboards.<sup>38</sup> Teachers actively include students in the class during an active learning environment.<sup>39</sup> Therefore, it can be argued that teachers' approach plays an important role in promoting an active learning environment especially when teaching Euclidean geometry.

## METHODOLOGY

In this research, a qualitative approach was selected as it facilitates the synthesis and analysis of common themes and trends in the perspectives of the participants. This method provided students with the opportunity to share comprehensive accounts of their opinions and viewpoints on their comprehension of Euclidean Geometry concepts, as per Creswell's guidelines.<sup>40</sup>

For this study, the case study research design was deemed appropriate for its qualitative mode of inquiry. This empirical approach allows for a deep exploration of a contemporary phenomenon, specifically Euclidean geometry, within its real-world context. By matching observed events with

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Saidat M. Adeniji et al., "Effect of Mastery Learning Approach on Senior School Students' Academic Performance and Retention in Circle Geometry.," *International Journal of Instruction* 11, no. 4 (2018): 951–62; Edmore Mukamba and Chipso Makamure, "Integration of GeoGebra in Teaching and Learning Geometric Transformations at Ordinary Level in Zimbabwe," *Contemporary Mathematics and Science Education* 1, no. 1 (July 30, 2020): ep20001.; Robert B.Armah, Primrose O.Coffie, and Christopher A. Okpoti, "Investigating the Effect of van Hiele Phase-Based Instruction on Pre-Service Teachers' Geometric Thinking," *International Journal of Research in Education and Science*, (2018), 314–30; Mohammad A. Al-Khateeb, "The Extent of Mathematics Teacher's Awareness of Their Students' Misconceptions in Learning Geometrical Concepts in the Intermediate Education Stage," *European Scientific Journal, ESJ* 12, no. 31 (2016): 357.; Eric Machisi, "Grade 11 Students' Reflections on Their Euclidean Geometry Learning Experiences," *Eurasia Journal of Mathematics, Science and Technology Education* 17, no. 2 (, 2021): em1938.

<sup>30</sup> Pierre M van Hiele, "Structure and Insight: A Theory of Mathematics Education," (No Title), 1986.

<sup>31</sup> Alex and Mammen, "A Survey of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the Van Hiele Theory."

<sup>32</sup> Lee Shulman, "Knowledge and Teaching: Foundations of the New Reform," *Harvard Educational Review* 57, no. 1(April 1, 1987): 1–23.

<sup>33</sup> Rahmiati et al., "The Effectiveness of Flipbook-Based e-Modules in Increasing Student Creativity in Nail Art Subject in Higher Education," *Jurnal Pendidikan Vokasi* 13, no. 2 (2023); Hilary Scandrett, "Using Geoboards in Primary Mathematics: Going... Going... Gone?," *Australian Primary Mathematics Classroom* 13, no. 2 (2008): 29–32.

<sup>34</sup> Mandlenkosi R. Sibiya, "The Effect of Geoboard Use on Learners' Motivation for Learning of Geometry Theorems," *International Journal of Sciences and Research* 75, no. 6/1 (2019).

<sup>35</sup> Rahmiati et al., "The Effectiveness of Flipbook-Based e-Modules.," Scandrett, "Using Geoboards in Primary Mathematics..."

<sup>36</sup> Sibiya, "The Effect of Geoboard Use on Learners' Motivation for Learning of Geometry Theorems."

<sup>37</sup> Imdad Ali, Bhagawati Simarekha, and Sarmah Jnanjyoti, "Performance of Geometry among the Secondary School Students of Bhurbandha CD Block of Morigaon District, Assam, India," *International Journal of Innovative Research and Development* 3, no. 1 (2014).

<sup>38</sup> Keith Jones, "Issues in the Teaching and Learning of Geometry," *Aspects of Teaching Secondary Mathematics* (Routledge, 2003), 137–55.

<sup>39</sup> Kelsey Hood Cattaneo, "Telling Active Learning Pedagogies Apart: From Theory to Practice," *Journal of New Approaches in Educational Research (NAER Journal)* 6, no. 2 (2017): 144–52.

<sup>40</sup> J. David Creswell, "Mindfulness Interventions," *Annual Review of Psychology* 68, no. 1 (January 3, 2017): 491–516.

theoretical predictions, this method can effectively explain outcomes. Hollweck and Yin define a case study as an empirical inquiry that investigates a contemporary phenomenon (the 'case') in depth and within its real-world context.<sup>41</sup>

To gather information from 15 mathematics learners in three secondary schools in the Eastern Cape, face-to-face interviews and an interpretive method were utilized. The interpretive paradigm was chosen based on the participants' attitudes and beliefs, as the researcher aimed to examine their understanding of geometrical concepts. This approach prioritizes the richness of the study context and allows participants to share their own thoughts and feelings about Euclidean geometrical concepts.<sup>42</sup>

A purposeful sampling technique was used to select respondents who were most likely to provide useful and relevant information. According to Kelly, Bourgeault, and Dingwall, this method is effective in discovering and choosing cases that make the most of limited research resources.<sup>43</sup> By using deliberate sampling, researchers were able to gather a wealth of information from their data. Purposeful sampling allowed them to extract a great deal of information and describe the major impact of their findings on the population. The study focused on van Hiele's theory, factors that hinder students' understanding of geometric concepts, and strategies to support their understanding.

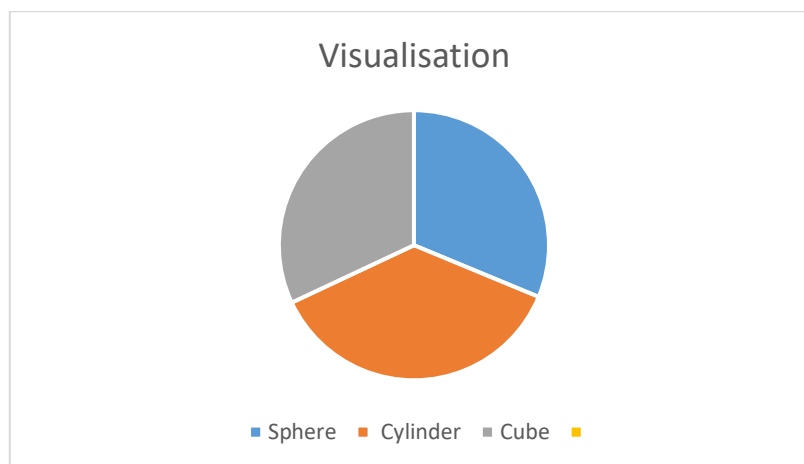
## PRESENTATION OF RESULTS

### *Learners' Performance in terms of the van Hiele Levels*

Throughout the interviews, the questions were classified according to the van Hiele levels of geometry cognition necessary to respond to them. The initial question tasked learners with identifying three geometric shapes depicted in Figure 2, aligning with the first tier of Van Hiele's framework. This stage demands students to accurately recognize geometric shapes by their physical properties.



**Figure 1:** Problem 1 Students were asked to identify the three geometric shapes that were depicted in Figure 2.



**Figure 2:** Students' Figure 1 performance in terms of Level 1: Visualisation

<sup>41</sup> Trista Hollweck, "Robert K. Yin. (2014). *Case Study Research Design and Methods (5th Ed.)*," *Canadian Journal of Program Evaluation* 30, no. 1 (March 2015): 108–10, <https://doi.org/10.3138/cjpe.30.1.108>.

<sup>42</sup> Husam Helmi Alharahsheh and Abraham Pius, "A Review of Key Paradigms: Positivism VS Interpretivism," *Global Academic Journal of Humanities and Social Sciences* 2, no. 3 (2020): 39–43.

<sup>43</sup> Susan E Kelly, I Bourgeault, and R Dingwall, "Qualitative Interviewing Techniques and Styles," *The SAGE Handbook of Qualitative Methods in Health Research* 19 (2010): 307–26.

From the figure above participants' responses show that the majority of participants answered the question about shapes correctly. However, a few students struggled with identifying the first and last shapes with precision. Specifically, some students referred to the first shape as a "spherical" or "circle" based on Figures 2, 3, 4, 5, and 6. In contrast, some students erroneously labelled the third shape as a "Rhombus" instead of a "cube".



Figure 2: Student S1's solution to Problem 1



Figure 3: Student S2 answer for Problem 1



Figure 4: Student S3 answer for Problem 1



Figure 5: Student S4 answer for Problem 1



Figure 6: Student S5 answer for problem 1

During the research, an interview was conducted to gain valuable insight into the challenges facing students. These challenges became apparent in the student's responses during the interview process. For example, when queried about why student S2 used the word 'circle' to describe the initial shape, the answer was simply that *"the shape was circular."* The researcher followed up by inquiring whether 'ring' or 'sphere' were also appropriate descriptors, but the learner was unsure and did not provide a definitive answer.

**Student S7 answer for problem 1:**

Sphere Cylinder Cube

**Students’ responses for Figure 8 which is based on Level 1: Analytic**

The questions asked during the interviews were categorized based on the van Hiele degrees of geometry cognition required to answer them. The first question, which matched the second level of Van Hiele's framework, required students to name twenty various geometric shapes shown in Figure 8. At this level, students must reliably identify geometric shapes based on their physical characteristics.

In problem 2 students were asked to identify the geometric shapes according to prisms, pyramids, cuboids, and cubes that were depicted in Figure 8.

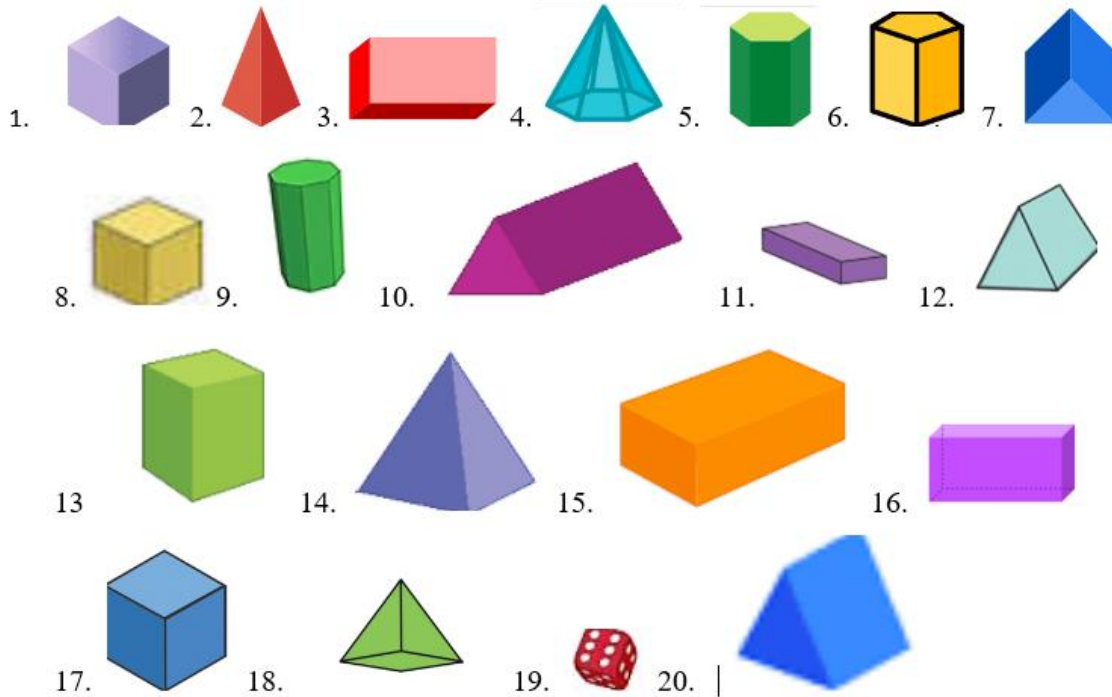


Figure 8: Geometric figures categorised according to Prisms, Pyramids, Cuboid, Cube

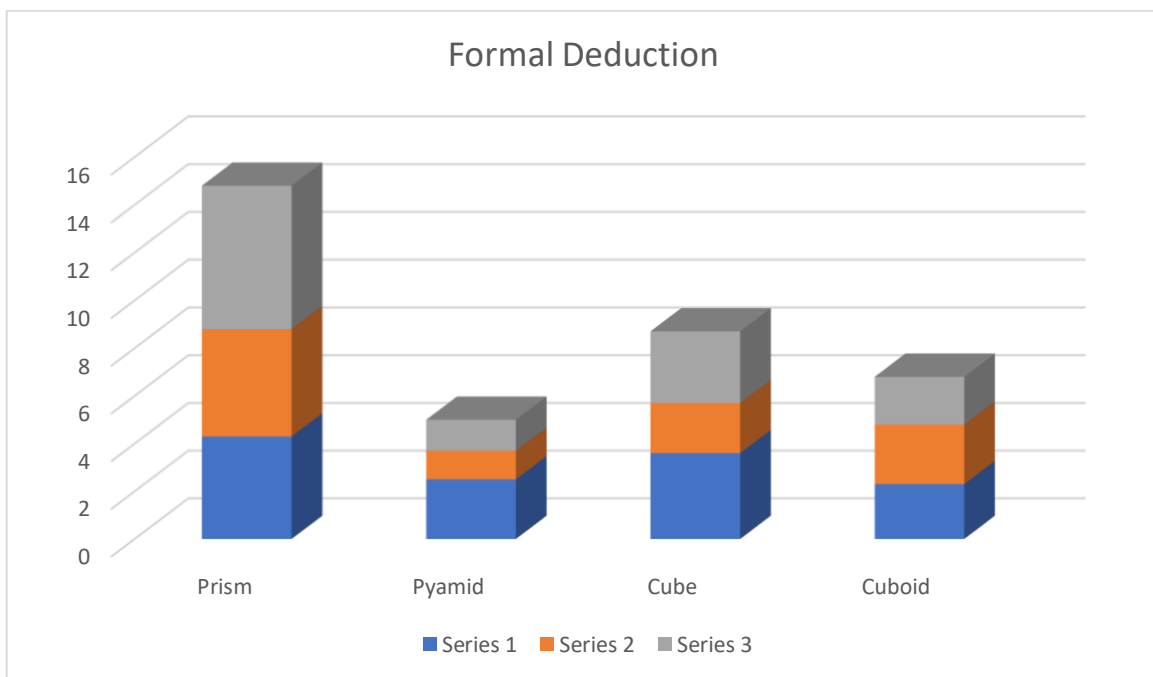


Figure 9: Students’ performance shown in the above Figure according to van Hiele’s Analytic level

It can be seen from Figure 9 that most of the participants correctly identified prisms concerning shapes in Figure 8. A few students, meanwhile, had trouble accurately distinguishing the first and last shapes. Some students specifically called the first shape a square.

When asked about Shape 3 according to Level 1 which is Analytic, the student responded by saying "The question was unclear, Doc. I observed two shapes within Shape 3 - one being a rectangle, but I am uncertain about the second." It is evident from the answers that some students struggled with identifying the shapes.

### **Student's answer for Problem 2**

Learners were asked according to their analytical level. Some could identify geometric figures by their properties but could not see interrelationships or understand definitions. As shown in Figure 10 students were required to categorize different geometric shapes in space, including pyramids, prisms, cubes, and cuboids.

Regarding problem 2, shapes 1, 2, 3, 4, 8, 13, and 15 were correctly recognized by their properties. However, when questioned about why shapes 1, 8, and 17 were categorized as cubes, different explanations were given by the learners. One participant said, "A cube is a three-dimensional shape with length, breadth, and height." Another participant explained, "A cube has equal sides." For Problem 2, Student S12 gave their answer. Out of all the students, only 5 identified shape 5 as a prism and were asked to explain their reasoning. Their justifications varied, with some stating that "a prism has two identical and parallel bases on one side" and another participant saying that "prisms have two bases." Another mentioned that "all faces of a cube are squares."

### **Student S3's answer for Problem 2**

According to Participant 6, shape 1 is a rhombus. When asked for justification, the participant explained that "all sides of the shape are equal and also form squares; all sides have equal dimensions."

### **Student S11's answer for Problem 2**

During an interview, Student 11 identified Shape 1 as a rhombus and defined it with four equal sides and two pairs of parallel lines. For Problem 2, Student S12 correct answers. Out of all the students, only 7 identified shape 6 as a prism and were asked to explain their reasoning. Their justifications varied, with some stating that "a prism has two identical and parallel bases on one side" and other students saying that "prisms have two bases," "A prism has two parallel bases on one side," while others simply stated that "a prism has two bases."

### **Student S13's answer for Problem 2**

When asked to identify shapes numbered 7, 9, 10, 12, and 13, many students correctly categorized them as prisms. One student even explained that "a prism has two bases that are the same shape and size, with all other sides being rectangular." However, some students classified them as cuboids.

Some students classify shapes as either cubes or prisms, but the correct answer is a prism. However, some students categorize it as a cuboid. For shape 13, most students had the correct answer (prism). Even if students correctly categorized the shape, they could not explain why a shape belongs to a particular category as opposed to others, as shown in the interview excerpt in Figure 8.

### **Student's answer for Problem 2 interview**

Based on the interview, students tended to categorize shapes by physical appearance rather than formal definition. For example, students identified shape #3 as a cube because it looked like one. However, some students had difficulty recognizing shapes from different angles, such as mistaking shape #15 for a cuboid based on the front view in image #16.

Students can classify geometric figures based on properties, improving class inclusion and meaningful definitions. On this level, students were asked according to the third level which is an informal deduction where learners begin to use reasoning to develop new ideas based on what they already know. For example, they can explain logically why squares are a special kind of rectangle. They do not need to

know how to measure angles. However, they can learn to use a corner of a piece of paper as a ‘right-angle checker’.

ABCE is a parallelogram  $\angle C_1 = 90^\circ$ . Give the sizes of fewer angles  $A_1$ ,  $A_2$ ,  $C_2$ , and  $C_3$  giving reasons for your answers.

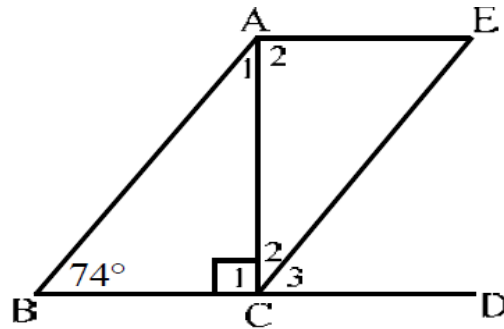


Figure 10: Problem 3 posed to students according to the fourth level: Formal Deduction

On the above question, most students demonstrated a clear understanding of angle properties and were able to calculate the size of angle  $A_1$ . They held the belief that the sum of interior angles in a triangle equates to 180 degrees. The following excerpt is taken from the interview conducted with the learners.

**An interview with learners regarding question 3 resulted in the following:**

*Researcher:* When you added, you said they must give  $180^\circ$ . Can you please explain the reason why you responded in that way?

**Learner LS1:** *During our lesson on geometric shapes, I recall learning that the sum of interior angles in a triangle equals  $180^\circ$ . For example, in triangle ABC, if angles B and C are  $74^\circ$  and  $90^\circ$  respectively, angle A1 must equal  $16^\circ$  degrees. Another participant shared that their teacher taught them that adjacent angles are supplementary, meaning they add up to 180 degrees. During our lessons on geometric shapes, I recall learning that the sum of interior angles in a triangle equals  $180^\circ$ . For example, in triangle ABC, if angles B and C are  $74^\circ$  and  $90^\circ$  degrees respectively, angle A1 must equal  $16^\circ$  degrees. Another participant shared that their teacher taught them that adjacent angles are supplementary, meaning they add up to 180 degrees.*

Almost all the learners in the interview said that  $\angle A_2 = 90^\circ$ . When they were asked what the reason was to say that angle  $A_2$  is equal to  $90^\circ$ . Different answers from learners were displayed.

**LS1:** “This angle is a right angle, and a right angle is equal to  $90^\circ$ ”.

**LS2:** “Right angle is equal to  $90^\circ$ ”.

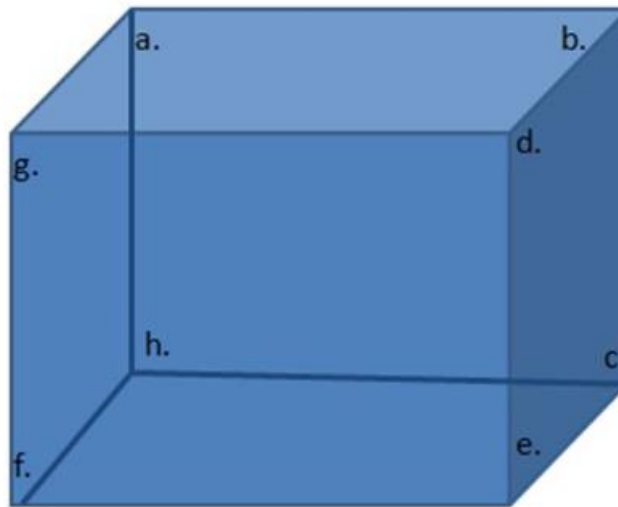
**LS3:** “Triangle ACE is a right-angled triangle and  $\angle A_2 = 90^\circ$ ”.

LS4, LP5, LP6, and LP7 responded that when two lines are cut by a transversal line, the pairs of alternate angles are equal; therefore, angle  $A_2 = 90^\circ$ . LS10 suggested that these angles are also vertically opposite angles. About angle  $C_2$  most of the learners said angle  $C_2$  is equal to  $16^\circ$ . When they were asked what the reason behind their answers was, LS1 said, “Alternate angles are equal.”

In the same vein, LS8 said, that when two parallel lines are cut by a transversal line the pairs of parallel lines are equal.” Most learners said that  $\angle C_3 = 74^\circ$  but they have different reasons, for example, LS1, LS3, LS6, and LS9 reason was, “corresponding angles are equal. Whereas other participants came up with a different reason, Angle  $C_3 = 74^\circ$  and supported that by saying that, “Angles on a straight line are equal to 180.”

The analysis also revealed that learners struggled with items involving class inclusion problems. Class inclusion is a property of geometric figures whereby a shape is composed of two triangles, and angle  $A_1$  is not adjacent to any other two interior angles of triangle ABC.

**For Problem 4,** The learners were assessed on their deductive abilities and can now construct proofs using definitions, axioms, theorems, and converses. This enables them to identify geometric properties like parallel and skewed lines on a cube (as shown in Figure 11). This assessment corresponds to the fourth level of Van Hiele's model.



*Figure 11: Problem 4 students were asked according to the formal deduction which is the fourth level of van Hiele's theory*

In a Formal Deduction, learners establish theorems within an axiomatic system. They recognize the difference between undefined terms, definitions, axioms, and theorems. They can construct original proofs.

**Problem 4**

*The following responses are some of the sample students' answers to Problem 4.*

**Interviewer:** Can you explain your reason for categorizing GA and FH as parallel lines?

**LS3:** When asked about parallel lines their responses were like this, "Initially, I was unsure of the shape's classification. However, upon comparing it to other shapes, I determined that it closely resembled GA and FH as parallel lines because they never meet. Although the shape is not two straight lines, I still considered it parallel because these lines never intersect each other".

Another participant responded by saying line GF is also parallel to DE, even these lines can never meet.

**LS5:** "Line GD and FE are parallel lines because parallel lines can be defined as two lines in the same plane that are at equal distances from each other and never meet. They can be both horizontal and vertical".

**LS6:** "The teacher taught that parallel lines never intersect, no matter how far you try to extend them in any given direction, and I conclude that lines BC and AH are parallel lines."

**Student S5's answer for Problem 4**

For Problem 4, most of the students correctly answered the questions in part (i) and part (ii). Students could list the pairs of parallel and skewed lines correctly, even though the answer had been given not quite complete (see Figure 12). After doing an in-depth interview, it revealed that students failed to identify other pairs because they depended on the visual appearances of the shape shown in the picture. For example, the lines AB and GD were not identified as parallel lines.

Regarding Problem 4, S6's answer showed some difficulty in correctly identifying the pairs of skew lines in part (iii). After interviewing the students, it was found that many of them misunderstood the term "skew" in "skew lines." They mistakenly believed it referred to the condition where two lines intersected like a regular line.

#### **Student's Answer to Problem 4**

For the fourth problem, students were asked to choose the skew lines among the diagram above. The question was made based on the fourth level of Van Hiele where the students were expected to build logical arguments in proving the skew lines.

Many students did not answer the question about "skew lines" correctly due to misunderstandings or misinterpretations. Some answers were incorrect, such as GF and BC, but some students answered correctly for lines GF and HC. During interviews, students struggled to explain their reasoning, indicating a reliance on rote memorization in geometry rather than deep understanding.

**LP3:** Similarly, this learner identified lines AB and CE as skew lines. When asked why he said so she replied by saying, "Skew lines are non-parallel lines in space that do not intersect".

**LP4:** "Skew lines have to be non-coplanar, which means that parallel lines must be found in more than one plane".

**LP5:** Gave the correct answer when saying that lines GA and DE are skew lines and supporting her answer by adding that "skew lines are two lines that do not intersect and are not parallel."

#### **Learners were asked about the impediments that they face while attempting to understand geometry.**

**LP1:** "I struggle with visualizing shapes from different angles and translating between diagrams and symbols.

**LP2:** "Geometry seems too abstract and disconnected from reality."

**LP3:** "It is difficult to grasp geometrical concepts like area and perimeter when complicating formulas and diagrams get in the way."

**LP4:** "Geometry is different from nearly everything you work on in mathematics because it is not purely focused on calculations."

**LP5:** "It is difficult to understand geometry because it mostly involves understanding how space works in relation to itself."

**LP6:** "Since our geometry teacher was redeployed to another school by the Department of Education, I have been feeling demotivated in class."

**LP7:** "Since my parents are illiterate, I do not have any help at home with my studies, but I do receive assistance from the school."

**LP8:** "My mathematics teacher always uses traditional methods and the textbook, which does not allow for individual attention due to noise and lack of support materials. "

#### **Ways to improve the teaching and learning of geometry**

##### **Teachers were asked about the strategies they used when teaching geometry**

**TP1:** "Sometimes if learners do not understand the concept (geometry), something I try my best to help them individually by making reference to the practical activities that I displayed on the blackboard."

**TP2:** "The teaching of mathematics should be practical to help learners understand concepts and enjoy the subject."

**TP3:** “Usually, I create lessons where learners can identify and analyze shapes with correct and incorrect examples.”

**TP4:** “I always urge my students to utilize technology and apply learning to real-world scenarios.”

**TP5:** “Despite a lack of Learner Teacher Support Material in our schools, I urge my students to engage in regular purposeful small group discussions.”

**TP6:** “In a geometric class, I usually use concrete examples such as pictures, charts, diagrams, etc.”

## DISCUSSION OF FINDINGS

### Student’s Difficulties

When trying to enhance a student's comprehension of Euclidean geometry, it is crucial to consider Van Hiele's five levels of geometry thinking.<sup>44</sup> However, this research discovered that numerous students lacked knowledge of Euclidean geometry at these five levels, resulting in subpar teaching and learning outcomes. Only a small number of educators met the acceptable teaching standards. The study revealed some challenges faced by senior secondary school students. Firstly, some of the learners were unable to identify the proper characteristics of geometric shapes. Analysis of the data reveals that students are still confused about naming and specifying properties of given geometric shapes. This demonstrates that students lack sufficient understanding of geometry concepts, particularly the formal definition of geometric shapes. Many students struggle with geometry, and this can result in difficulties when it comes to categorizing certain shapes. Research has shown that not only is geometry challenging for students to learn, but it can also be difficult for teachers to teach. This is because it involves practical activities, and many teachers lack the necessary subject content knowledge and teaching strategies to deliver the subject effectively. However, it is essential to focus on developing geometrical skills, as they are crucial for careers in construction work, architectural design, and engineering.<sup>45</sup> Bayuningsih has highlighted that learners often struggle with geometry because they do not fully understand the concepts.<sup>46</sup> Alex and Mammen's study also found that many students struggle with identifying geometric shapes based on their properties.<sup>47</sup> Furthermore, some students find it challenging to visualize specific geometric shapes, which can impede their ability to classify and identify shapes accurately. This is because visualization is crucial for understanding how shapes move and change. Students who lack this ability may rely on visual prototypes instead of formal definitions.

Hock et al and Gutiérrez's studies have shown that many students have difficulty visualizing three-dimensional objects on a flat surface, which can lead to confusion when it comes to identifying the properties of solid shapes.<sup>48</sup> Additionally, students may struggle with the language used in geometry, including specific symbols and terms. Some studies have identified this as a contributing factor to students' difficulties.

Finally, some students struggle with constructing logical structures to prove geometric statements. They may find it challenging to establish connections between different statements, and this can hinder their ability to provide reasoning and interpret answers. Asemani and Asiedu-Addo have also noted that some students struggle to extract necessary information from given data and draw conclusions.<sup>49</sup>

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<sup>44</sup> Hiele, “Structure and Insight: A Theory of Mathematics Education.”

<sup>45</sup> Bansilal and Ubah, “The Use of Semiotic Representations in Reasoning about Similar Triangles in Euclidean Geometry”; Alex and Mammen, “A Survey of South African Grade 10 Learners’ Geometric Thinking Levels in Terms of the Van Hiele Theory.”

<sup>46</sup> A S Bayuningsih, B Usodo, and S Subanti, “Problem Based Learning with Scaffolding Technique on Geometry,” *Journal of Physics: Conference Series* 1013 (May 2018): 012134, <https://doi.org/10.1088/1742-6596/1013/1/012134>.

<sup>47</sup> Alex, Jogymol, and Kuttikkattu J. Mammen. "Students’ understanding of geometry terminology through the lens of Van Hiele theory." *Pythagoras* 39, no. 1 (2018): 1-8.

<sup>48</sup> Tong Hock Tan et al., “Understanding the Primary School Students’ van Hiele Levels of Geometry Thinking in Learning Shapes and Spaces: A Q-Methodology,” *EURASIA Journal of Mathematics, Science and Technology Education* 11, no. 4 (August 25, 2015), <https://doi.org/10.12973/eurasia.2015.1439a>.

<sup>49</sup> Asemani, Asiedu-Addo, and Oppong, “The Geometric Augmented Reality for Junior High School Students .”

### **Student's level of thinking**

From the evaluation of student performance, it seems that a lot of learners struggled with identifying geometric shapes based on their visual appearance (Level 1). Some could describe the characteristics of given shapes (Level 2), but a few had difficulties providing accurate responses due to a lack of comprehension of certain geometry concepts, particularly specific terms. Moreover, a small number of students answered questions related to Level 3 but could not give a complete answer. This could be because they had limited knowledge of formal definitions or properties of shapes, relying mostly on visual prototypes. None of the students reached Level 4, as evidenced by their inability to provide proper reasoning and proof for Problem 4. According to Luneta's research, high school students should be operating between Levels 3 and 4, but none of the learners in this study attained Level 4 of Van Hiele. Less than half of the students were on Levels 2 and 3, with most of them on Level 1. This study also supports Hock et al.'s findings that students cannot advance to a higher level of thinking if they have not mastered the previous level.

### **RECOMMENDATIONS**

Based on Van Hiele's theory it is just not possible to reach higher levels of geometric development without mastering the initial levels first.<sup>50</sup> Teachers should therefore be encouraged to use this channel of communication to assist learners who are falling behind for whatever reason. This would be on a more individual basis and not necessarily for the whole group. If parents are unable to assist, it is recommended that the school ask other parents who have extra time to come into the school to spend time working with the learners who need extra assistance. Reading skills are frequently aided by this parent support approach. Similar techniques can be used to stress mathematics abilities, such as having a parent spend an hour in the classroom helping struggling students. Therefore, the problem of class size needs to be addressed on a systematic basis, especially in the foundation phase. Smaller classes would enable students and teachers to spend more one-on-one time together. In addition to greatly assisting in the scaffolding of geometry learning, this will also greatly assist in other learning.

As a result, it is too late to begin an intervention during the high school period, and the following suggestions are therefore made: More possibilities for adult-supervised play should be provided by foundation phase and intermediate phase teachers, especially when teaching geometry. Additionally, parents are urged to help their children study geometry. Instead of asking parents to teach concepts, it is preferable to use this time to play games that have been carefully chosen to improve learning. Many parents would be delighted to have the chance to raise their children, but frequently lack the knowledge to do so.

This can be a very practical choice for teachers to meaningfully include parents thanks to the usage of WhatsApp groups and straightforward voice or video instructions. It is also recommended that the curriculum should allow more time to teach geometry in the foundation phase syllabus. This will allow for time to deal more specifically with fundamental concepts such as disembodying, composing, decomposing, spatial visualization, and spatial orientation skills, so that teachers adhering to the curriculum are aware that these must be included. There is an imbalance in how much emphasis is placed on geometry in high school and how much attention is given in the foundation phase. There are 2 solutions to this imbalance. More time can be allocated to geometry in the early years or less time and emphasis must be placed on geometry at a high school level.

### **CONCLUSION**

According to the data gathered for this research, many learners have limited geometric understanding and lack formal or informal deduction skills, often only reaching the first or second level of van Hiele's model. More than one-third of the group could only reason at the Visual level, which means they could only see shapes as a whole and could not identify properties within a figure or the relationships between figures. The results also demonstrated issues with proving skills, which may help to explain why geometry is often seen as a challenging section of mathematics. The learners were not given enough opportunities to develop the necessary reasoning skills at the higher van Hiele levels. In addition, the participants had difficulties

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<sup>50</sup> Hiele, "Structure and Insight: A Theory of Mathematics Education."

with class inclusion reasoning and distinguishing between properties necessarily satisfied by a specific figure and those sufficient for a general figure to be part of the special figures class. In the researcher's opinion, the value of geometry cannot be understated, and it would be disadvantageous to a child's development to see it underplayed. This is because geometry is not only practical in terms of everyday functioning and certain vocational skills, but it trains the mind to see mathematics visually, to be methodical in reasoning and deductions, and to understand other areas in mathematics. It also provides parents and teachers with the opportunity to play with mathematics and help children develop a love for mathematics due to the variety of playful and visual activities that can be developed in geometry.

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