

Grade R Numerosity Levels and Gaps: A Case of South African Learners in the Eastern Cape



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ABSTRACT

Developing young learners' knowledge in number sense is a priority if the aim is to build a rich mathematical foundation for successful learning and future innovative careers. Capturing students' interests and motivation is crucial while mediating counting concepts. Literature directs practice on the important core concepts that are foundational in developing number sense. This paper therefore assessed how young South African children demonstrated number concepts before entering the reception class. The paper revealed the diverse knowledge attained by children in different settings before embarking on formal education. A qualitative analysis was employed using Clements and Sarama's learning trajectories as an analytical tool for children's counting progression. The findings indicated that children's start mathematically is uneven with either fully or partially attained number concepts, for example, one-to-one correspondence, counting on, cardinality, and equality. Furthermore, the findings revealed that a lack of particular numerical skills such as keeping track while counting, reciter, and sequential verbal counting beyond 10 impede learners' full realisation of understanding numerical concepts. The findings suggest pre-schooling stimulation that provides rich mathematical experiences and purposeful play towards the attainment of core foundational concepts. It was, therefore, recommended that Grade R Mathematics teachers intervene earlier during preschool to assist learners specifically from low socioeconomic backgrounds and mathematise learners' play and activities to enhance their semi-attained numerical abilities. This paper informs the mathematics education community of the need for future interventions and a curriculum designed to improve these skills in young children. This could have a positive impact on long-term academic success and close the numeracy gap which is existing in mathematics achievement between high and low-income children.

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INTRODUCTION

Research shows that South African learners' Mathematics and literacy performance has been standing at a comparatively lower level both regionally and internationally.¹ Furthermore, the performance gaps in Mathematics accentuated by socio-economic status are at the extreme.² Since the dawn of democracy,

¹ Mellony Holm Graven, "Poverty, Inequality and Mathematics Performance: The Case of South Africa's Post-Apartheid Context," *Zdm* 46 (2014): 1039–49.

² V. Reddy et al., *Beyond Benchmarks: What Twenty Years of TIMSS Data Tell Us About South African Education* (Cape Town, South Africa: HSRC Press, 2015).

South Africa has prioritized redressing inequalities that existed in education. Education is considered a potent tool for redressing persistent economic inequality. This paper discusses the Grade R learners' numerosity levels and gaps in the Eastern Cape province of South Africa. Below is a brief description of the South African education system.

In South Africa, the education system is pitched at different levels integrated into a National Qualifications Framework as outlined by the South African Qualification Authority Act (SAQA) of 1996. SAQA established a seamless system of education to adapt South African citizens of all ages at distinct instructional levels. The instructional levels are summarized below:

- **Pre-primary education:** an optional level for young children up to the age of six years, Grade R and any level below that. This level is commonly known as the Early Childhood Development (ECD) phase.
- **Primary education (grades 1 to 6):** for children between 6 and 12 years of age (older children are allowed to attend).
- **Junior secondary education (grades 7 to 9):** offered to children between 12 and 15 years of age. The first year of junior schooling is offered at the primary school level while the successive two years are offered in senior secondary schools.
- **Senior secondary education (grades 10 to 12):** for students between 16 and 20 years of age (older students are allowed to attend). The academic training or vocational and technical education and training are provided at this level. The general education programmes offered last for three years while technical programmes last for 2, 3, or 4 years.

Number sense dominates mathematical concepts across the school Mathematics curriculum; hence, it is important to understand it as a foundational building block for young learners. Nurturing number sense in young children must be associated with their interests that can only be identified through their play. Literature asserts that the diverse experiences of young children prior to formal education provide them with different numerical abilities.³ These diverse abilities play a significant role in children's development of numerical fluency while it is recognised that low performance is proven to be associated with limited numerical experiences. The latter is associated with the notion that some learners start with Numeracy knowledge gaps as compared to their peers.⁴ Aunio and Räsänen suggest four core numerical skills in numeracy development that are symbolic and non-symbolic number sense, understanding mathematical relations, counting skills, and basic skills in arithmetic that children need to attain.⁵ South Africa formalised the provision of Grade R/ reception class by including it in schools and making it universal. This implies that before Grade R, some children attend preschools, and others stay at home due to access and affordability constraints.

Based on the nature and diversity of preschool provision in South Africa, one cannot assume gaps and existing numeracy abilities from the young learners. Numerosity is considered a foundational mathematics skill often overlooked in early childhood education, this paper sheds light on the current state of numerosity levels and gaps in grade R. This paper reports on pre-Grade R learners' numerical abilities to identify the preschool and home stimulation influence on their numerosity as well as identify the learning gaps. The paper therefore responds to the following research question: How does preschool and home stimulation received by Grade R learners from low socio-economic backgrounds present itself in their numeracy observable abilities?

³ Geetha B Ramani and Robert S Siegler, "Reducing the Gap in Numerical Knowledge between Low-and Middle-Income Preschoolers," *Journal of Applied Developmental Psychology* 32, no. 3 (2011): 146–59; Pirjo Aunio and Pekka Räsänen, "Core Numerical Skills for Learning Mathematics in Children Aged Five to Eight Years – a Working Model for Educators," *European Early Childhood Education Research Journal* 24, no. 5 (September 2, 2016): 684–704, <https://doi.org/10.1080/1350293X.2014.996424>; Jill Cheeseman, Christiane Benz, and Yianna Pullen, "Number Sense: The Impact of a Measurement-Focused Program on Young Children's Number Learning," *Contemporary Research and Perspectives on Early Childhood Mathematics Education*, 2018, 101–27..

⁴ Aunio and Räsänen, "Core Numerical Skills for Learning Mathematics in Children Aged Five to Eight Years – a Working Model for Educators."

⁵ Aunio and Räsänen, "Core Numerical Skills for Learning Mathematics in Children Aged Five to Eight Years – a Working Model for Educators."

In the literature review section, the following subtopics are presented and discussed: subitising, importance of subitising, verbal/rote counting, one-to-one correspondence, cardinality principle, numerical spatial sense and spatial thinking skills.

LITERATURE REVIEW

The literature presented in this paper is focused on crucial foundational concepts of counting related to the interview items used in this paper. It is important to note that counting for young children requires both procedural skills as well as conceptual understanding and their interdependence on one another.⁶

Subitising

The term subitising was created by Piaget and described as the ability to rapidly identify the number of objects in a small group without counting them.⁷ Clements discusses subitising as an important skill that is innate in children for small numbers to three, and he labels this spontaneous subitising.⁸ Breive posits that three-year-olds can subitise up to three while the majority of adults can subitise all faces of dice without counting.⁹ Clements affirms that spontaneous subitising is not an indication that children can count but a precursor to counting. He encourages the development of conceptual subitising for meaningful counting.¹⁰ This process, innate to all humans, is typically known as perceptual subitising and forms an element of the preverbal number sense.¹¹ In short, perceptual subitising is recognising numerosity without using other mathematical processes.¹²

Importance of Subitising

Subitising assists children in linking numbers with actual objects or groups of objects a process called number conservation. Fuson proposes that young children may use perceptual subitising to make units of counting building on to their initial ideas of cardinality.¹³ It nurtures children's composition of numbers, for example, five is composed of three and two, it also becomes a foundational block for building part-part whole relationships that will eventually increase their understanding of addition and subtraction.

It is conceptual subitising that develops from children's patterning abilities to advance groups of numbers beyond five. This skill develops their arithmetic abilities.¹⁴ Baroody asserts that subitising is an essential skill for developing students' meaningful number sense.¹⁵ Different studies indicate number concepts that are reinforced by conceptual subitising such as equality, composition, decomposition, partitioning and addition.¹⁶

Verbal Counting/Rote Counting

Verbal counting/rote counting occurs intentionally or unintentionally when a student has memorised the numerical words chronologically.¹⁷ Children practise rote counting long before it makes any sense to them before they grasp that the numbers signify the quantity of something. This counting has a significant role in the development of the number sense of a learner at this level and it is a skill that needs to be mastered. In her study, Feza states that some five-year-olds who could not do verbal counting got stuck at counting objects because they had not yet mastered the sequence of numbers.¹⁸ In their attempt to do object

⁶ Faith H Sadler, "Help! They Still Don't Understand Counting.," *Teaching Exceptional Children Plus* 6, no. 1 (2009): n1.

⁷ J. Piaget, *The Child's Conception of Number* (London: Routledge & Kegan Paul, 1952).

⁸ Douglas H Clements, "Subitizing: What Is It? Why Teach It?," *Teaching Children Mathematics* 5, no. 7 (1999): 400–405.

⁹ S. Breive, "Processes of Mathematical Inquiry in Kindergarten" (University of Agder, 2019).

¹⁰ Clements, "Subitizing: What Is It? Why Teach It?"

¹¹ Charles R Gallistel and Rochel Gelman, "Non-Verbal Numerical Cognition: From Reals to Integers," *Trends in Cognitive Sciences* 4, no. 2 (2000): 59–65.

¹² Clements, "Subitizing: What Is It? Why Teach It?"

¹³ K. C. Fuson, "Research on Whole Number Addition and Subtraction," in *Handbook of Research on Mathematics Teaching and Learning*, ed. Douglas A. Grauw (New York: Macmillan Publishing Co., 1992), 243–75.

¹⁴ Clements, "Subitizing: What Is It? Why Teach It?"

¹⁵ Arthur J Baroody, "Counting Ability of Moderately and Mildly Handicapped Children," *Education and Training of the Mentally Retarded*, 1986, 289–300.

¹⁶ Fenna Van Nes and Jan de Lange, "Mathematics Education and Neurosciences: Relating Spatial Structures to the Development of Spatial Sense and Number Sense," *The Mathematics Enthusiast* 4, no. 2 (2007): 210–29.

¹⁷ Nosisi N. Feza, "Teaching 5- and 6-Year-Olds to Count: Knowledge of South African Educators," *Early Childhood Education Journal* 44, no. 5 (September 9, 2016): 483–89, <https://doi.org/10.1007/s10643-015-0736-z>.

¹⁸ Feza, "Teaching 5- and 6-Year-Olds to Count: Knowledge of South African Educators."

counting, their counting could not be accurate as they skipped some number words or went back to repeat others. Memorising sequential number words became a tool needed to be able to successfully engage in object counting. Clements, Clements and Sarama, and Wu suggest that counting games, counting songs, counting rhymes, and jumping games should be used to mediate sequential rote counting.¹⁹

Researchers found that learners' rote counting varies if they attain productivity via different routes in different languages, for example, if the language is of greater transparency or if the learners received lengthy and comprehensive rote learning in the early childhood stage.²⁰ This implies that language of instruction and exposure to deep learning activities on number senses play a prominent role in supporting learners' rote counting.

Other scientific studies have shown that young children understand the concept of numbers before they enter the school system, for example, a child can give five candles; can count verbally in the correct sequence, i.e. one, two, three.....and can use this sequence in counting objects, family members, friends, etc; and comparing the numerical size of separate sets (** vs. ** or 3 vs. 2).²¹ These basic number skills are the basis for learning basic arithmetical skills for learners in their earliest years in school.²²

One to One Correspondence

Clements and Sarama refer to this level of counting as object counting. It is a progressive skill that comprises counting each object in a set once, and only once with one touch per object.²³ Furthermore, it refers to a relationship between two sets such that every component of the first set matches one, and only one component of the second set.²⁴ One-to-one correspondence empowers children to comprehend the number system and affords an elementary foundation to build on.²⁵ Grasping one-to-one correspondence nurtures structure for meaningful counting although it is not easy for children to understand.²⁶

Cardinality Principle

In describing the cardinal principle, Nikoloska describes it as identifying the last number used in counting to determine the cardinality of the set.²⁷ For example, if a child counts some objects and is asked the question "How many?" they should be able to repeat the last number if they have seized the principle of cardinality. Wynn argues that being able to give the cardinal number does not mean the child understands the cardinality principle as it might be possible that it is a "last word rule" that is used by the child.²⁸ Piaget, and Gelman and Gallistel indicate that cardinal numbers quantify sets, for example, three pigs, five friends, two apples and so on.²⁹ Baroody and Gatzke define cardinality as the child's acknowledgement that the last number counted includes the whole group of items counted.³⁰ Wynn then proposes the use of diverse tasks to determine understanding of the cardinality principle.³¹ Hence, Clements and Sarama hypothesised developmental progressions in counting leading to cardinality that is verbal counting followed by sequential verbal counting, and object counting which has skills such as tracking for accuracy.³² These are skills that are building blocks for counting before meaningful counting of objects

¹⁹ Douglas H Clements, "Supporting Young Children's Logo Programming," *Computing Teacher* 11, no. 5 (1984): 24–30; Douglas H Clements, Julie Sarama, and Candace Joswick, "Learning and Teaching Geometry in Early Childhood," in *Special Issues in Early Childhood Mathematics Education Research* (Brill, 2022), 95–131..

²⁰ Mellony Graven and Robyn Jorgensen, "Unexpected Outcomes of a Family Mathematics Story-Time Program.," *Mathematics Education Research Group of Australasia*, 2018.

²¹ Tuire Koponen, Kaisa Aunola, and Jari-Erik Nurmi, "Verbal Counting Skill Predicts Later Math Performance and Difficulties in Middle School," *Contemporary Educational Psychology* 59 (2019): 101803.

²² Tuire Koponen et al., "Counting and Rapid Naming Predict the Fluency of Arithmetic and Reading Skills," *Contemporary Educational Psychology* 44–45 (January 2016): 83–94, <https://doi.org/10.1016/j.cedpsych.2016.02.004>.

²³ D. H. Clements and J. Sarama, *Learning and Teaching Early Math: The Learning Trajectories Approach* (New York: Routledge, 2009).

²⁴ Robert Reys et al., *Helping Children Learn Mathematics* (John Wiley & Sons, 2014).

²⁵ Nunes, T., Bryant, P., Evans, D., Bell, D., Gardner, S., Gardner, A., & Carragher, J. N. (2007). The Contribution of Logical Reasoning to the Learning of Mathematics in Primary School. *British Journal of Developmental Psychology*, 25, 147-166

²⁶ Feza, "Teaching 5- and 6-Year-Olds to Count: Knowledge of South African Educators."

²⁷ Ana Nikoloska, "Development of the Cardinality Principle in Macedonian Preschool Children," *Psihologija* 42, no.4(2009): 459–75.

²⁸ Karen Wynn, "Children's Understanding of Counting," *Cognition* 36, no. 2 (1990): 155–93.

²⁹ Piaget, *The Child's Conception of Number*; Gallistel and Gelman, "Non-Verbal Numerical Cognition: From Reals to Integers."

³⁰ Arthur J Baroody and Mary R Gatzke, "The Estimation of Set Size by Potentially Gifted Kindergarten-Age Children," *Journal for Research in Mathematics Education* 22, no. 1 (1991): 59–68.

³¹ Wynn, "Children's Understanding of Counting."

³² Douglas H Clements and Julie Sarama, "Learning Trajectories in Early Mathematics—Sequences of Acquisition and Teaching," *Encyclopedia of Language and Literacy Development* 7 (2009): 1–6.

that will then lead to cardinality. Therefore, cardinality can be mediated through repeated routines suggested by Fuson and Wynn such as three bananas, “one, two, three”.³³ Mix confirms that children’s attainment of the cardinal principle leads to recognition of equivalence relations and construction of number representations at the age of three to four.³⁴

Numerical Spatial Sense

A numerical spatial sense is the ability to rapidly estimate quantities of items without being confused by their spatial positioning. Anobile et al. describe the numerical spatial sense as an ability to make intrinsic linkages between Mathematics and spatial awareness.³⁵ In counting, the spatial numerical sense is linked with one-to-one correspondence as children need the skill for them to be able to develop cognitively as stated by Piaget. Sophian elaborates this association using two rows with an equal number of objects, with the other row stretched longer than the other by increasing the distance between them.³⁶ This implies that the student who understands one-to-one correspondence will count both rows and see that they have the same number of objects whereas students with no understanding of one-to-one correspondence will not be able to see them as equivalent.

Current studies on numerical, spatial, and executive function (EF) skills reveal that each plays critical and independent roles in the learning and performance of Numeracy in learners.³⁷ Research hypothesises that spatial visualisation has a significant role in assisting learners with the ability to model, simulate and form mathematical relationships and transformations, for example, the ability to decompose 15 as the unit of 10 and 5.³⁸ Moreover, these are the same spatial reasoning capacities that help learners with the grounded understanding of various symbolic number relations, learning and representation of symbolic Mathematics in general.³⁹

Spatial Thinking Skills

Scientific studies have shown that there is an association between spatial thinking and performance across a wider breadth of mathematical topics.⁴⁰ Spatial skills contribute positively to mathematical topics, for example, arithmetic,⁴¹ word problems,⁴² geometry⁴³ and algebra.⁴⁴ Spatial thinking also forms a foundation for highly advanced Mathematics such as function theory, mathematical logic, computational mathematics⁴⁵ and comparing, i.e., which digit is numerically larger than the other between 7 and 2. It is further directly linked with spatial visualisation skills, such as mental rotation.⁴⁶ Findings related to spatial

³³ K. Fuson, *Children’s Counting and Concept of Number* (New York, NY: Springer-Verlag, 1988).

³⁴ Kelly S Mix, “Surface Similarity and Label Knowledge Impact Early Numerical Comparisons,” *British Journal of Developmental Psychology* 26, no. 1 (2008): 13–32.

³⁵ Giovanni Anobile et al., “Spatial but Not Temporal Numerosity Thresholds Correlate with Formal Math Skills in Children.,” *Developmental Psychology* 54, no. 3 (March 2018): 458–73, <https://doi.org/10.1037/dev0000448>.

³⁶ Catherine Sophian, “Early Developments in Children’s Understanding of Number: Inferences about Numerosity and One-to-One Correspondence,” *Child Development*, 1988, 1397–1414.

³⁷ Scott A Gilmore, “Immunity Disorders: The Conflict of Foreign Official Immunity and Human Rights Litigation,” *Geo. Wash. L. Rev.* 80 (2011): 918.

³⁸ Zachary Hawes et al., “Relations between Numerical, Spatial, and Executive Function Skills and Mathematics Achievement: A Latent-Variable Approach,” *Cognitive Psychology* 109 (2019): 68–90.

³⁹ G. Lakoff and R. E. Núñez, *Where Mathematics Comes from: How the Embodied Mind Brings Mathematics into Being* (New York, NY: Basic Books, 2000).

⁴⁰ Hawes et al., “Relations between Numerical, Spatial, and Executive Function Skills and Mathematics Achievement: A Latent-Variable Approach.”

⁴¹ Minna Kyttälä and Juhani E Lehto, “Some Factors Underlying Mathematical Performance: The Role of Visuospatial Working Memory and Non-Verbal Intelligence,” *European Journal of Psychology of Education* 23 (2008): 77–94.

⁴² Mary Hegarty and Maria Kozhevnikov, “Types of Visual–Spatial Representations and Mathematical Problem Solving.,” *Journal of Educational Psychology* 91, no. 4 (1999): 684.

⁴³ Ana R Delgado and Gerardo Prieto, “Cognitive Mediators and Sex-Related Differences in Mathematics,” *Intelligence* 32, no. 1 (2004): 25–32.

⁴⁴ Tammy Daun Tolar, Amy R Lederberg, and Jack M Fletcher, “A Structural Model of Algebra Achievement: Computational Fluency and Spatial Visualisation as Mediators of the Effect of Working Memory on Algebra Achievement,” *Educational Psychology* 29, no. 2 (2009): 239–66.

⁴⁵ Wei Wei et al., “Cognitive Correlates of Performance in Advanced Mathematics,” *British Journal of Educational Psychology* 82, no. 1 (2012): 157–81.

⁴⁶ Arnaud Viarouge, Edward M Hubbard, and Bruce D McCandliss, “The Cognitive Mechanisms of the SNARC Effect: An Individual Differences Approach,” *PloS One* 9, no. 4 (2014): e95756.

thinking like these inform academics in Mathematics education to have a keen interest in advancing learners' spatial thinking skills as they have a positive impact on Mathematics performance.

A study conducted by Thomas established that kindergarten children had a great deal of knowledge about shapes and matching shapes before instruction began which is contrary to the assumption that teachers and curriculum writers in some countries hold that learners in early childhood classrooms have little or no knowledge of geometric figures.⁴⁷ Thus, it is because of the later assumptions that many classrooms exhibit limited geometry instruction. Clements, Swaminathan, Hannibal, and Sarama's study exploring learners' spatial understanding found that learners identified circles quite accurately: 92%, 96%, and 99% for 4-, 5-, and 6-year-olds respectively.⁴⁸ Most of them described the circle as round if they described them at all. Thus, it was easy to recognise the circle, and on the other hand, it was relatively difficult to describe its properties. The study further revealed that learners were able to match the shapes to a visual prototype provided.

METHODOLOGY

The study employed a qualitative approach guided by the case study design. The case study design was opted for as it allows researchers to explore a real-life or contemporary bounded system (a case) over time, through comprehensive, in-depth data collection involving numerous sources of information and reports a case explanation and case themes.⁴⁹ Clinical interviews as employed by Van Hiele and Clements to identify the thinking levels of children are administered to pre-Grade R learners to identify their numeracy levels of thinking. A qualitative analysis was employed using Clements and Sarama's learning trajectories as an analytical tool for counting progression.⁵⁰ The use of these learning trajectories was influenced by the support this theory receives in researching numeracy globally.⁵¹ This theory influences the conceptualisation of this study as well as the early childhood mathematics literature. A thematic report is presented as the findings of the paper.

This paper reported on data from a three-year NRF-funded study selecting only five learners that represent the majority, based on their responses. Three items from the clinical interview instrument were selected to identify the numerical skills and numerical knowledge learners possess. A qualitative analytical approach was used to analyse learners' responses using the learning trajectory theory of Clements and Sarama.⁵² The three selected items bring diverse ways of exploring learners' numeracy skills.

- Item 1 is free play with counters (unifix cubes).
- Item 2 is a quantity recognition item.
- Item 3 is a matching item for symbolic numbers with objects.

PRESENTATION OF FINDINGS

The biographical information from the data revealed that out of five students composed of three boys and two girls, three attended preschool and two did not attend preschool at all. Four themes merged from the analytic analyses and are presented below.

Diverse levels of one-to-one correspondence

To test learners' counting skills they were given two sets of counters as shown in Figure 1 below:

⁴⁷ B. Thomas, "An Abstract of Kindergarten Teachers' Elicitation and Utilization of Children's Prior Knowledge in the Teaching of Shape Concepts" (New York: School of education, health, nursing, and arts professions, New York University, 1982); Clements, Sarama, and Joswick, "Learning and Teaching Geometry in Early Childhood."

⁴⁸ Douglas H Clements et al., "Young Children's Concepts of Shape," *Journal for Research in Mathematics Education*, 1999, 192–212.

⁴⁹ John W Creswell, *Educational Research: Planning, Conducting, and Evaluating* (W. Ross MacDonald School Resource Services Library, 2013).

⁵⁰ Clements and Sarama, *Learning and Teaching Early Math: The Learning Trajectories Approach*.

⁵¹ B. Harrison and D. Peterson, "Developing Math Skills in Early Childhood," *Mathematica Policy Research Issue Brief*, 2017, <http://files.eric.ed.gov>.

⁵² Clements and Sarama, *Learning and Teaching Early Math: The Learning Trajectories Approach*.



Figure 1: shows two sets of counters

Researcher: Please count the counters

The learners involved in this study demonstrated the ability to count the given objects, but they demonstrated that they are at different developmental levels. For example,

L1: lacked object counting skills, after counting 1 to 5, then doubled counted the last two objects. Instead of 7 objects, he ended up counting 9.

L2: counted accurately but was challenged by cardinality. For example, when asked to count the two sets of counters as shown above, L2 counted the first five and then moved on to the second group to 10 but later said that there were 12 counters.

L3: During free play, all L3 did was count each unifix he picked to 10, then decided to connect them together. Surprisingly, when asked to count the two groups together he could not count on from the first 5 counters. He started from one again and instead gave a cardinal response of one group. An indication that he was not able to count on from 5 to 10.

Spatiality and Quantity Indistinct

The two sets of counters shown in Figure 2 below were given to the learners to test their spatiality and quantity distinct skills they were asked to recognise the equivalence.



Figure 2: shows two sets of counters

Researcher: Which ones are many? How do you see there are many?

Researcher: Please count them.

Researcher: How many bottle tops together?

Spacing of the items influenced their quantity recognition even after counting only one of them recognized the equivalence.

L2 and **L3** identified the first image as the one with many counters while **L1** identified the second image as one with many counters.

When learners assessed their responses with counting and realised that both groups had 5 objects only **L2** said “ziyalingana” meaning they were equal. Both **L1** and **L3** still remained with their first choices.

Recognition and non-recognition of numeral symbols

Learners matching abilities of objects to numerals were also shown at diverse levels such as the ability to read numeral symbols and the inability to read the numeral symbols.

L1 did not recognise any numeral symbols.

L2 identified all the numerals and matched them with objects representing their number accurately.

L3 managed to count objects correctly, could not recognise 5 and 3 but could match 1, 2 and 4 correctly. L4 was able to count from 1 to 6 in English, and counted the given objects and the number of dots on the play cards, but could not write the symbol 6 when asked to do so.

Turning now to the length of a shoe and an envelope, L5 responded as described below.

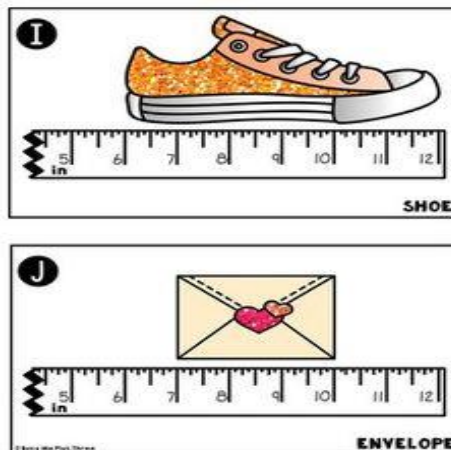


Figure 3: showing envelope lengths

Researcher: Mom says this shoe ends at six, but my brother says it ends at 12 what do you say?

Researcher: Look at the envelope, where should it end?

When asked where the shoe and the envelope ended on the number line (as shown above), L5 said that they ended at 3 and 2, respectively.

Bilingual Code-switching

L1: Counted in isiXhosa his home language and language of instruction but went further to bigger numbers and switched to English number names.

L2: Counted in English numerals but uses some isiXhosa mathematical terminology for sense-making.

L3: Counted in English numerals then used isiXhosa to communicate his strategy.

L4: Counted from 1 to 6 in English yet the question was asked in isiXhosa (the learner's home language).

Geometric and spatial thinking

L4 was asked to pick up the shapes that looked exactly like the ones drawn below from the container.

The task below required learners to identify each of the shapes and to take out one that looks like each one of them from the container.

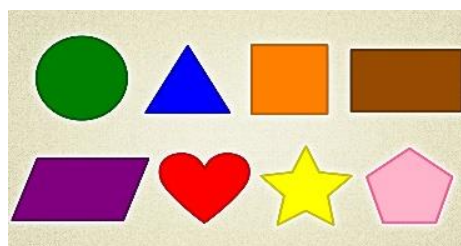


Figure 4: Geometric shapes

Researcher: Kweza zinto zininzi ubudlala ngazo ungakwazi ukumatanisa ezifana nazo apha? Zeziphi zithathe? Zifana ngantoni? (Which manipulatives are the same as these, from the ones you were playing with?)

L5 could not select the shapes as required instead only selected the red triangled shapes from the container.

DISCUSSION

The findings highlighted that there are several diverse numerical abilities that learners demonstrated but that are not fully established such as one-to-one correspondence, counting on, cardinality and equality. Secondly, a lack of numeric skills impeded the full realisation of the concepts mentioned, those skills are

keeping track while counting, reciter, and sequential verbal counting beyond 10. Bilingualism automatically occurs between English and isiXhosa and mathematical language used is from both languages, for example, L1, L2, L3 and L4's way of using more than one language revealed that most learners do not completely rely on one language to understand Mathematics concepts, even the second language (English) plays a role in their learning. This finding supports Clements and Sarama's argument which states that for a learner to count properly, the learner must have a progressive skill whereby each object in a set is touched and counted once.⁵³ The progressive skill is not limited to a certain language of instruction; it is applied in Mathematics despite the language of instruction being used.

Compared to international literature, learner participants in this study entered the Grade R class with haphazard numerical abilities, and none of the concepts were fully realised. Although they are supposed to be counters that can produce numbers, at this stage sub-concepts are not attained. These findings are like those of learners in Macedonia who enter school without preschool experience.⁵⁴ They also were at different levels of attaining cardinal principles. Research has consistently demonstrated that these gaps envisage long-term consequences as they predict achievement scores in Grades 1,3,5, 8, and high school.⁵⁵ This indicates the importance of intervening earlier during preschool to assist learners specifically from low socioeconomic backgrounds. Hence, the proposal of mathematising learners' games and activities enhances their semi-attained numerical abilities.⁵⁶

This study has shown that although learners are in the same grade, they exhibit different levels to recognise and read numeral symbols, for example, L1's inability to recognise any numerical symbols confirms previous research findings that students count intentionally or unintentionally because of songs and games they play while learning Mathematics.⁵⁷ At this level more is done verbally than in writing as a result students struggled with the writing of mathematical symbols. L2 was able to identify and recognise numerals and matched them with objects representing their number accurately. L3 was able to count objects correctly, could not recognise 5 and 3 but could match 1, 2 and 4 correctly.

L4 was able to count from 1 to 6 in English, and counted the given objects and the number of dots on the play cards, but could not write the symbol 6 when asked to do so.

Discussion Summary

The findings of this study have highlighted diverse numerical abilities learners demonstrated that are not fully established, for example, one-to-one correspondence, counting on, cardinality and equality. Secondly, the lack of some numerical skills impedes learners' full realisation of the concepts mentioned. Those skills are keeping track while counting, reciter, and sequential verbal counting beyond 10. When learners have no solid foundation in numerosity skills, fail to grasp more complex mathematics concepts later in their academic life. This can lead to frustration and disengagement from the subject.

Learners in the same grade exhibit different levels of recognition and reading numerical symbols. For this reason, there is a need for intervention earlier during preschool to assist learners specifically those from low socioeconomic backgrounds to acquire mathematical skills. Schools need to fully support learners as they come from different backgrounds, some do have access to resources while others do not have any access to the resources for learning mathematics, particularly those from low-income homes.

RECOMMENDATIONS

Based on the findings of this study, the researchers recommend the following:

- Grade R Mathematics teachers intervene earlier during preschool to assist learners specifically from low socioeconomic backgrounds and mathematise learners' play and activities to enhance their semi-attained numerical abilities.
- Grade R teachers and policymakers must target their efforts to close the numeracy gap that exists between young children and ensure that all learners have the opportunity to succeed regardless of their family background.

⁵³ Clements and Sarama, *Learning and Teaching Early Math: The Learning Trajectories Approach*.

⁵⁴ Nikoloska, "Development of the Cardinality Principle in Macedonian Preschool Children."

⁵⁵ Ramani and Siegler, "Reducing the Gap in Numerical Knowledge between Low-and Middle-Income Preschoolers."

⁵⁶ Clements and Sarama, *Learning and Teaching Early Math: The Learning Trajectories Approach*.

⁵⁷ Feza, "Teaching 5- and 6-Year-Olds to Count: Knowledge of South African Educators."

- Numerosity is a foundational mathematics skill often overlooked in early childhood education, based on the numerosity levels and gaps in grade R, this paper informs the mathematics education community to develop interventions and design the curricula to improve these skills in young children. This can positively impact learners' long-term academic success and close the numeracy gap which is existing in mathematics achievement between high and low-income children.

CONCLUSION

This study has assessed how young South African children demonstrated number concepts before entering the reception class. This study's findings suggest that there is a need for more investment in early childhood education, particularly around the area of numerosity. In addition, it is important to teach the area of numerosity early, as numerosity develops in the early years and has a lasting impact on later mathematics achievement. Teaching numerosity for conceptual understanding assists in closing the gap that exists between high and low-income learners. The closure of such a gap assists mathematics teachers in closing the gap between high and low-income status learners in the achievement of mathematics later.

LIMITATIONS

These research findings are transferrable to other contexts to strengthen the study and confirm the findings. It is advisable to interpret these findings with caution as they are based on the data collected from the same cultural group of learners who speak the same home language. The replication of the same research with a different cultural group may yield different findings.

AREAS OF FURTHER STUDIES

The present researchers suggest that further studies can be conducted on:

- Exploring causes of learners' diverse numerical abilities.
- Investigating how teachers teach number sense in diverse classrooms.
- Exploring the effectiveness of specific interventions on the teaching and learning of numerosity
- Investigating the relationship between numerosity and other mathematics skills

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CONFLICT OF INTEREST

No potential conflict of interest was reported by the authors.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author, S.M. Chiphambo. The data is not publicly available due to [restrictions, as the study involved minors and the data contains information that could compromise their privacy].

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