E-Journal of Humanities, Arts and Social Sciences (EHASS)
ISSN – Online 2720-7722 | Print 2821-8949
Volume 4 Issue 12 November 2023 Special Issue pp 126 - 142
Available online at: https://noyam.org/journals/ehass/
DOI: https://doi.org/10.38159/ehass.202341212

Teaching Fractions in the Intermediate Phase: A Neuroscience Approach

Matshidiso Mirriam Moleko [1] & Percy Sepeng [2]
[1] Department of Curriculum Studies, University of South Africa, Tshwane, South Africa
[2] Department of Natural Sciences Teaching, Sol Plaatje University, Kimberley, South Africa

ABSTRACT
Fractions are often problematic for learners to fully grasp. Teachers must therefore determine various approaches for teaching fractions to reinforce understanding. More so, teachers need to determine strategies that cater for a diverse learner population and promote meaningful learning. In the quest to find a possible solution to address this challenge, a qualitative study was conducted to demonstrate a neuroscience approach to teaching fractions in the intermediate phase through the application of the universal design for learning (UDL) principles. The universal design for learning through neuroscience was used as a theoretical framework to guide the study. Data was collected through a series of video-recorded observations and audio-recorded focus group discussions. The Free Attitude Interview (FAI) technique was used to guide the facilitation of group discussions. Four intermediate-phase teachers were purposefully selected to participate in the study. Data was analysed using content analysis and UDL guidelines. The findings of the study reflect the UDL-based practices that were used to optimise and promote meaningful learning of fractions. The study thus recommends the application of UDL principles to promote access, build strong foundations, and assist learners to internalise "fractions" (content). The study further recommends the application of UDL to promote meaningful learning.

Correspondence
Matshidiso Mirriam Moleko
Email: molekmm1@unisa.ac.za

Publication History
Received 26th August, 2023
Accepted 13th October, 2023
Published online:
5th December, 2023

Keywords: *Fractions, Intermediate Phase, Neuroscience Approach, Teaching*

INTRODUCTION
Fractions are infamously difficult for learners to comprehend and often pose ongoing pedagogical challenges to mathematics teachers.[1] They form part of the mathematics curriculum (Curriculum and Assessment Policy Statement - CAPS) in the intermediate phase and are crucial to learners' future understanding of related concepts such as comparative/proportional reasoning that are essential not only for reinforcing a deeper understanding of mathematics but also to support daily activities. Although fractions are crucial for learners' future understanding of the related concepts, they often present challenges that warrant teachers' attention to ensure that a strong foundation is laid.[2]

The difficulties that are presented by "fractions" are often identified and realised across all levels of education, starting from the early primary years.[3] Several reasons are attributed to these difficulties, especially in primary schools.[4] For instance, Getenet and Callingham aver that greater mathematics cognitive processes,

[1] Dianne Siemon et al., *Teaching Mathematics: Foundations to Middle Years* (University Of Tasmania, 2015).
[2] Seyum Getenet and Rosemary Callingham, "Teaching Interrelated Concepts of Fraction for Understanding and Teacher's Pedagogical Content Knowledge," *Mathematics Education Research Journal* 33 (2021): 201–21.
[3] Susan B Empson and Linda Levi, "Extending Children's Mathematics: Fractions and Decimals," *Mathematics Education* 27, no. 4 (2011): 403–34.
[4] Joan Moss and Robbie Case, "Developing Children's Understanding of the Rational Numbers: A New Model and an Experimental Curriculum," *Journal for Research in Mathematics Education* 30, no. 2 (1999): 122–47.

including proportional and spatial reasoning, underpin the understanding of fractions.[5] Often, learners lack proportional and spatial reasoning skills, making it difficult for them to grasp the concept of fractions.[6] Hackenberg and Lee avow that the limited comprehension of specific aspects of the different meanings of the fraction concept adversely affects the ability of learners to work with the concept effectively.[7] In their study, Siemon et al. found fractions to be complex for learners to grasp because they are generally used to characterise a relationship between numbers, rather than an absolute number.[8] According to Chamane, for many learners, the understanding of fractions is characterised by knowledge of rote procedures, which are often inaccurate, rather than by the concepts underlying those procedures.[9]

Despite these challenges, teachers' efforts to promote the effective teaching of fractions are observable. For instance, manipulatives and real-life problems are often used to enable learners to understand the fraction concept.[10] These strategies were found to be interesting to the learners and helped them construct their own knowledge about the fraction concept.[11] Researchers have discovered that learners who are actively engaged in mathematical activities that use manipulatives develop an understanding of the concepts. Thus, the gap between the abstract and concrete is bridged, since the manipulatives enable them to visualise the mathematical concepts.[12] Although the manipulatives can help learners understand mathematical concepts and strengthen their problem-solving abilities, research shows that learners who learn with manipulatives can become too reliant on the object and context, and as a result, have difficulty transferring their knowledge to new contexts.[13] The mere use of manipulatives does not necessarily guarantee the learner's understanding of fractions (concept).[14] Too much use of manipulatives can slow down learners' mathematical development and these materials can become a crutch that inhibits them from developing more sophisticated ways of solving problems.[15]

Storytelling, which is another commonly used teaching strategy, proves to be effective in terms of teaching fractions by providing a meaningful context that attracts learners' interest while at the same time, making learning a pleasant process.[16] The use of the Concrete-Pictorial-Abstract (CPA) strategy was also found to have a positive effect on learners'mathematical conceptual understanding (MCU).[17]

While the above-mentioned strategies assist in making the fraction concept easy to understand, they do not provide an all-encompassing type of pedagogy which considers the concurrent use of representation, engagement, action and expression strategies to accommodate the needs and abilities of all learners. This eliminates unnecessary hurdles in the learning process and improves the learning experience for all. Hence, the study aims to propose a neuroscience approach to teaching fractions in the intermediate phase. A neuroscience approach requires the application of representation strategies to activate the recognition brain network that makes it possible for learners to recognise and understand the content. The approach further necessitates the use of action and expression strategies to activate the strategic brain network that enables learners to demonstrate

[5] Getenet and Callingham, "Teaching Interrelated Concepts of Fraction for Understanding and Teacher's Pedagogical Content Knowledge."
[6] Wenke Möhring et al., "Spatial Proportional Reasoning Is Associated with Formal Knowledge about Fractions," *Journal of Cognition and Development* 17, no. 1 (2016): 67–84.
[7] Amy J Hackenberg and Mi Yeon Lee, "Relationships between Students' Fractional Knowledge and Equation Writing," *Journal for Research in Mathematics Education* 46, no. 2 (2015): 196–243.
[8] Siemon et al., *Teaching Mathematics: Foundations to Middle Years*.
[9] Cynthia Nonhlanhla Chamane, "Exploring Teachers' Experiences of Teaching Fractions in Grade 6 in the Curriculum and Assessment Policy Statement: A Case Study of One Rural School in Ndwedwe Circuit." (2016).
[10] Methuseli Moyo and France M Machaba, "Grade 9 Learners' Understanding of Fraction Concepts: Equality of Fractions, Numerator and Denominator," *Pythagoras* 42, no. 1 (2021): 602.
[11] Emilie A Naiser, Wendy E Wright, and Robert M Capraro, "Teaching Fractions: Strategies Used for Teaching Fractions to Middle Grades Students," *Journal of Research in Childhood Education* 18, no. 3 (2003): 193–98.
[12] Kira J Carbonneau and Scott C Marley, "Instructional Guidance and Realism of Manipulatives Influence Preschool Children's Mathematics Learning," *The Journal of Experimental Education* 83, no. 4 (2015): 495–513; Julie P Jones and Margaret Tiller, "Using Concrete Manipulatives in Mathematical Instruction.," *Dimensions of Early Childhood* 45, no. 1 (2017): 18–23.
[13] Matthew Boggan, Sallie Harper, and Anna Whitmire, "Using Manipulatives to Teach Elementary Mathematics.," *Journal of Instructional Pedagogies* 3 (2010).
[14] Lindiwe M. Mntunjani, Stanley A. Adendorff, and Sibawu W. Siyepu, "Foundation Phase Teachers' Use of Manipulatives to Teach Number Concepts: A Critical Analysis," *South African Journal of Childhood Education* 8, no. 1 (November 21, 2018), https://doi.org/10.4102/sajce.v8i1.495.
[15] Boggan, Harper, and Whitmire, "Using Manipulatives to Teach Elementary Mathematics."
[16] Charalambos Lemonidis and Ioanna Kaiafa, "The Effect of Using Storytelling Strategy on Students' Performance in Fractions.," *Journal of Education and Learning* 8, no. 2 (2019): 165–75.
[17] I Purwadi, I Sudiarta, and I Nengah Suparta, "The Effect of Concrete-Pictorial-Abstract Strategy toward Students' Mathematical Conceptual Understanding and Mathematical Representation on Fractions.," *International Journal of Instruction* 12, no. 1 (2019): 1113–26.

their thinking processes in varied ways. Lastly, the approach requires varied engagement strategies to activate the affective brain network that helps sustain effort and persistence.[18]

The study thus contributes to advancing the literature on the teaching and learning approaches/methods in mathematics. It proposes a new neuroscience-based approach for teaching fractions in the intermediate phase. The findings of the study are expected to enlighten intermediate phase teachers and inspire them to transform their practices by shifting from traditional methods to approaches that cater for a diverse learner population. The implications of this study are positioned in debates on promoting meaningful teaching and learning, as highlighted by the results of some international benchmark assessments, such as *Trends in International Mathematics and Science Study* (TIMSS). Based on the above, therefore, the study seeks to respond to the following research question: How can the neuroscience teaching approach be implemented to optimise and promote meaningful learning of fractions in the intermediate phase?

LITERATURE REVIEW

Knowledge of fractions epitomises the learner's elementary essential venture into conceptual mathematics.[19] According to Ndalichako, fractions help to facilitate learners' grasp and mastery of mathematical themes such as percentages, ratios and decimal numbers.[20] The knowledge of fractions not only allows learners to perform operations but also affords them a base to cope with related complex topics such as rates, percentages, slopes, and various other topics in mathematics.[21]

The "Fractions" topic has proven to be significant despite being consistently challenging to teach and learn.[22] Research shows that fractions are a challenge in primary schools worldwide. Booth and Newton and Torbeyns, Schneider, Xin and Siegler note that the challenges encountered with fractions are common, even among teachers from diverse countries.[23] These authors further note that the learners' challenges with fractions are far from being over and will continue to exist over time. According to Lin et al., the challenges experienced with fractions are often aggravated by teachers who demonstrate poor conceptual understanding of fraction arithmetic.[24] This poor conceptual understanding makes it difficult to teach fractions productively.

Fazio and Siegler note that teachers who use unproductive practices to teach fractions cause learning to be challenging.[25] As a result, learners cannot develop conceptual knowledge of fractions as expected. The unproductive teaching of fractions causes learners not to understand the concepts. According to Hoxha and Vula, one of the factors that makes teaching fractions challenging is that learners do not regard fractions as numbers.[26] They (learners) deal with the numerator and denominator as separate numbers rather than as a single number.[27] Learners who detach a numerator from a denominator often tend to memorise the terms and forget to understand the whole concept of a fraction.[28]

According to Vitoria, Fauzi and Ananda, the implementation of fractional theory is demonstrable in everyday life, which features discussions regarding distances and ages, length measurements, weight, and volume.[29] Therefore, this necessitates using real-life objects to ensure that learning is embedded in real life.[30] Also, the

---

[18] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]* (Wakefield, MA: National Center on AEM, 2018).

[19] Bojan Lazić et al., "On the Teaching and Learning of Fractions through a Conceptual Generalization Approach," *International Electronic Journal of Mathematics Education* 12, no. 3 (2017): 749–67.

[20] Joyce Lazaro Ndalichako, "Analysis of Pupils' Difficulties in Solving Questions Related to Fractions: The Case of Primary School Leaving Examination in Tanzania," *Creative Education* 4, no. 09 (2013): 69.

[21] Ji-Won Son and Sharon L Senk, "How Reform Curricula in the USA and Korea Present Multiplication and Division of Fractions," *Educational Studies in Mathematics* 74 (2010): 117–42.

[22] Peter Gould, Lynne Outhred, and Michael Mitchelmore, "One-Third Is Three-Quarters of One-Half," in *Annual Meeting of the Mathematics Education Research Group of Australia, Australia*, 2006.

[23] Julie L Booth and Kristie J Newton, "Fractions: Could They Really Be the Gatekeeper's Doorman?," *Contemporary Educational Psychology* 37, no. 4 (2012): 247–53; Joke Torbeyns et al., "Bridging the Gap: Fraction Understanding Is Central to Mathematics Achievement in Students from Three Different Continents," *Learning and Instruction* 37 (2015): 5–13.

[24] Cheng-Yao Lin et al., "Preservice Teachers' Conceptual and Procedural Knowledge of Fraction Operations: A Comparative Study of the U Nited S Tates and T Aiwan," *School Science and Mathematics* 113, no. 1 (2013): 41–51.

[25] L. K. Fazio and R. S. Siegler, "Teaching Fractions," in *Educational Practices Series* (Geneva: International Academy of EducationInternational Bureau of Education, 2011).

[26] M. Hoxha and E. Vula, "Improving the Teaching of Fractions through Collaborative Research," *Albanian Journal of Educational Studies* 2, no. 2 (2014): 4–18.

[27] Hoxha and Vula, "Improving the Teaching of Fractions through Collaborative Research."

[28] Hoxha and Vula, "Improving the Teaching of Fractions through Collaborative Research."

[29] Linda Vitoria, Fauzi Fauzi, and Nadia Ananda, "Students' Performance in Solving Problems Involving Fractions," *Proceedings of AICS-Social Sciences* 7 (2017): 707–14.

[30] Mirriam Matshidiso Moleko and Mogege D Mosimege, "Teachers' and Learners' Experiences for Guiding Effective Teaching and Learning of Mathematics Word Problems," *Issues in Educational Research* 30, no. 4 (2020): 1375–94.

fractional theory requires the use of tangible objects or manipulatives to make content understandable. Such a practice is essential to ensure learners develop the necessary acquaintance with fractions and good algebraic skills.[31]

Teaching of Fractions

Fazio and Siegler advocate for the use of real-life situations to teach fractions meaningfully.[32] These researchers regard the form of teaching that links the concepts learned in class with the outside reality as beneficial. According to Mwakapenda, using real-life situations in mathematics can increase learner interest in mathematics and promote critical reflection and active participation.[33] Wood and Millichamp also advocate for increasing learner participation to encourage the learning of fractions.[34] The use of games while teaching fractions was useful in integrating knowledge and skills gained by learners. Such a practice is regarded as highly motivational to learners, resulting in considerable attentiveness and involvement with mathematics.[35] Torbeyns et al. stress the need for teachers to be knowledgeable about their learners' fractions challenges to adapt their teaching to suit their competency levels.[36] These researchers further allude that teachers should be intentional about selecting specific resources and be certain about the scaffold they can provide learners.

THEORETICAL FRAMEWORK

Universal Design for Learning (UDL-cognitive-neuroscience teaching framework) was adopted as a theoretical framework underpinning the study to frame a pedagogy for teaching fractions in the intermediate phase to facilitate meaningful learning. UDL was coined by David Rose and his associates at the Centre for Applied Special Technology.[37] It is a framework based on neuroscience research that has identified three primary neurological networks that impact learning.[38] The three neurological networks are recognition, strategic, and affective, networks.[39] According to neuroscience research, for meaningful learning to occur, these three brain networks need to be activated.

A Recognition Network (RN) is a specialised network that enables humans to sense and assign meaning to patterns they see. In the context of teaching and learning, it enables students to recognise and comprehend information, ideas, and concepts.[40]

A Strategic Network (SN) is a specialised brain network that deals with how learners organise, plan, and initiate purposeful actions in class.[41]

An Affective Network (AN) is a specialised network used to assess patterns and assign them emotional significance.[42] This network enables students to engage with tasks, learning, and the world around them.

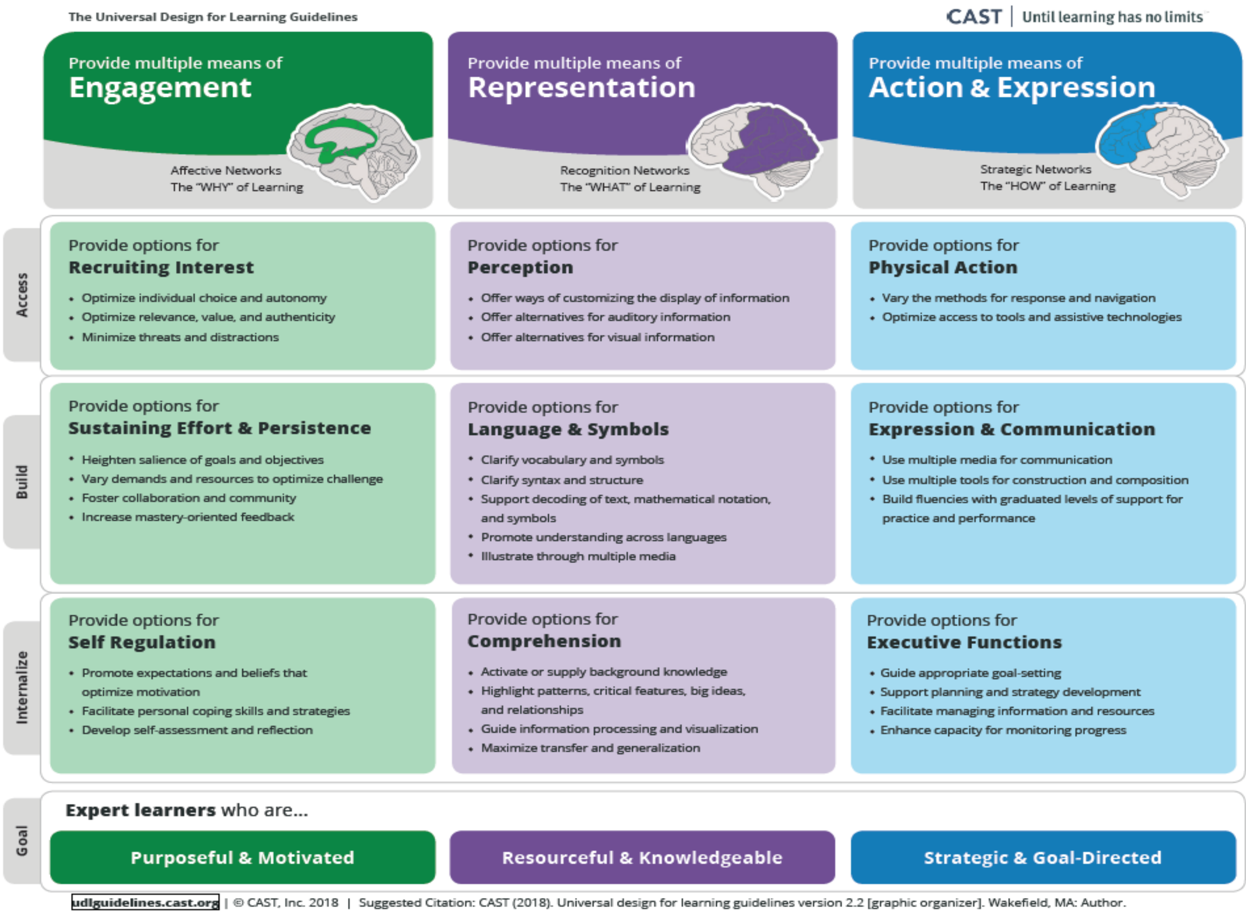
According to neuroscience research, successful teaching and learning involve all three networks simultaneously.[43] For the recognition network to be activated, multiple representations must be used. To activate the strategic network, multiple means of action and expressions must be used. The activation of the affective

---

[31] Julie L Booth, Kristie J Newton, and Laura K Twiss-Garrity, "The Impact of Fraction Magnitude Knowledge on Algebra Performance and Learning," *Journal of Experimental Child Psychology* 118 (2014): 110–18.
[32] Fazio and Siegler, "Teaching Fractions."
[33] Willy Mwakapenda, "Using Everyday Experiences in Teaching Mathematics: A Case Study of Change in Malawian Classrooms," *Journal of the Southern African Association for Research in Mathematics, Science and Technology Education* 5, no. 1 (2001): 53–64.
[34] E Kay Wood and Pat Millichamp, "Changing the Learning Ethos in School," *Journal of In-Service Education* 26, no. 3 (2000): 499–515.
[35] Doreen Drews and A Hansen, "Do Resources Matter in Primary Mathematics Teaching and Learning," *Using Resources to Support Mathematical Thinking, Primary and Early Years*, 2007, 19–31.
[36] Torbeyns et al., "Bridging the Gap: Fraction Understanding Is Central to Mathematics Achievement in Students from Three Different Continents."
[37] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]* (Wakefield, MA: National Center on AEM, 2011).
[38] John T Bruer, "Points of View: On the Implications of Neuroscience Research for Science Teaching and Learning: Are There Any? A Skeptical Theme and Variations: The Primacy of Psychology in the Science of Learning," *CBE—Life Sciences Education* 5, no. 2 (2006): 104–10.
[39] Elizabeth M Dalton, "Beyond Universal Design for Learning: Guiding Principles to Reduce Barriers to Digital & Media Literacy Competence.," *Journal of Media Literacy Education* 9, no. 2 (2017): 17–29.
[40] Susan Barteaux, "Universal Design for Learning.," *BU Journal of Graduate Studies in Education* 6, no.1(2014):50–54.
[41] Kavita Rao and Grace Meo, "Using Universal Design for Learning to Design Standards-Based Lessons," *Sage Open* 6, no. 4 (2016): 2158244016680688.
[42] Kathleen A Boothe et al., "Applying the Principles of Universal Design for Learning (UDL) in the College Classroom.," *Journal of Special Education Apprenticeship* 7, no. 3 (2018): n3.
[43] Richard Guy and Bruce Byrne, "Neuroscience and Learning: Implications for Teaching Practice," *Journal of Experimental Neuroscience* 7 (2013): 39.

network requires the use of multiple means of engagement.[44] Table 1 shows the UDL cognitive neuroscience framework for teaching. This study uses this framework in conjunction with the thematic analysis technique to make sense of empirical data and to frame a neuroscience-based approach for teaching the fraction concept in the intermediate phase.

Table 1: Universal Design for Learning (UDL) cognitive neuroscience framework



METHODOLOGY

A series of observation lessons and focus group discussions were conducted to generate the data in this qualitative case study. Four educators from two intermediate phase schools (2 educators per school) in Motheo District, in the Free State Province, were selected as participants in this study. Besides having long teaching experience (12–18 years), these teachers were highly respected for the good work that they were doing in terms of the teaching of mathematics in the intermediate phase. They had previously undergone the Universal Design for Learning (UDL) training on how to implement its principles in mathematics classrooms. Subsequently, they were selected as facilitators, responsible for training other teachers in the UDL concept. The observation lessons and focus group discussions were recorded. An observation form was used to note/identify the practices used in class. A free attitude interview technique was employed to facilitate the conversations during the focus group discussions. According to Meulen-Busken, the application of the FAI technique requires researchers to ask one comprehensive question to initiate the discussion.[45] In line with this, therefore, one comprehensive question (open-ended) was asked to initiate the discussion, followed by clarity-seeking questions from the researcher. The participants deliberated as in their day-to-day conversations. Field notes were also taken in the process to ensure that no important information or facts were omitted.

[44] Matshidiso M Moleko, "Multiple Means of Engagement Strategies for Maximising the Learning of Mathematics in Pandemic-Regulated Classrooms," *International Journal of Learning, Teaching and Educational Research* 20, no. 8 (2021): 68–90.
[45] I Meulenberg-Buskens, "Free Attitude Interview Technique," *London: Unpublished Notes*, 2011.

The qualitative content analysis technique was used to analyse and interpret the content of textual data, such as the interview transcripts and verbal communication (dialogue) in this study. The technique was used to identify patterns, concepts, and themes within the data to gain insight into the meaning and context of the content. The technique was deemed appropriate for the study to answer the "why", "how", or "what" questions, thus making it easy to identify "what" the teacher was teaching; "how" they taught the concept, and "why" they taught the concept in the manner that they did. The nature of the content analysis technique in terms of the questions it answers, resonates with the nature of the neuroscience approach to teaching, which requires teaching to speak to these three questions. In this study, data was categorised, organised and analysed using the three UDL principles (MMR, MMAE and MME). These principles were used to make sense of the data, thus reflecting the UDL practices used to inform a neuroscience-based approach for teaching fractions in the intermediate phase.

To ensure the trustworthiness and credibility of the study, the focus group discussions and lesson observations were audio and video recorded. Notes were also taken in the process to ensure that no important information or facts were missed. The use of multiple data collection instruments (focus group discussions and observations) enabled the researcher to triangulate the data. Multiple data instruments not only assisted in generating rich data but also made it possible to compare and distinguish practices when verbally explained, as opposed to when practically observed in class. Lastly, the involvement of the four teachers, who were deemed suitable participants for the study due to their expertise and experience of teaching mathematics in the intermediate phase and their knowledge of universal design for learning (neuroscience-based teaching framework), ensured the credibility and trustworthiness of the study.

FINDINGS AND DISCUSSIONS

The subsequent sections highlight the findings and provide discussions based on the generated data pertaining to the teaching of fractions. The UDL framework was used to analyse and make sense of the data. The sections reflect the UDL practices used to inform a neuroscience-based approach for teaching fractions in the intermediate phase.

Making Fractions (content) Accessible and Comprehensible through Multiple Representations

Often, the problem with fractions starts because learners are not given the time to develop a sound understanding of what fractions are. If they do not fully grasp what ¾ represents, then they cannot be expected to work with this fraction and learn how it relates to other numerical values. Teachers, therefore, must use strategies that aid learners' understanding of fractions. Multiple means of representation (MMR) is a UDL principle which is associated with the recognition brain network. The principle necessitates content to be represented/presented in varied formats to make it accessible and comprehensible. When applied, MMR provides learners with a variety of ways in which they can access and engage with the learning materials and information. The lesson extract below shows how Ms. Potsane (pseudonym) applied MMR to make content (fractions) accessible and comprehensible. All names of participants are pseudonyms to ensure anonymity.

Lesson Presentation Extract

Ms. Potsane: *Before we can start let me ask you some questions; what is sharing? Why do we have to share? When do we share? Does anyone have an answer to these questions? ….What is sharing?...(L1)*

Themba: *Sharing means diving a cake into small pieces and giving equal pieces to the people [Ms Potsane: writing down the notes; sharing - dividing]………………………………………………………………………….(L2)*

Ms. Potsane: *Thank you Themba. Why do we share? ….yes, Pakiso!...........(L3)*

Pakiso: *I think we share because sharing is caring. We want everybody to have a piece of cake……………………………………………………………………......(L4)*

Ms. Potsane [smiling]: *Thank you Pakiso. When do we share? Yes Thembi..(L5)*

Thembi: *Ma'm, we share when we have a small cake so that everyone can have something…………………………………………………………………………..(L6)*

Ms. Potsane: *I am glad you know what sharing is and why we have to share. Good!...(L7)*

Ms. Potsane: *Now let us look at this cake* ***(see Fig 1)****! If I take one piece from it and give it to Thabo, then it means I took one out of how many pieces in total?...(L8)*

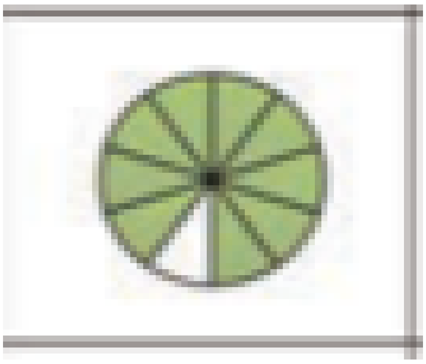


Figure 1: *1/10 cake demonstration*

Class: *One out of ten pieces…………………………………………………......(L9)*

Ms. Potsane: *Good! One out of ten pieces. Now when we write one out of ten pieces we write it in this manner 1/10. If I give away two pieces, what will be the answer and how should I write the answer? Yes Karabelo?.................(L10)*

Karabelo: *The answer will be two out of ten…………………………………..(L11)*

Ms. Potsane : *Good!..(L12)*

Ms. Potsane: *….one out of ten (writing on the board) 1/10, we call it a fraction meaning a small portion of a bigger cake….one is called a numerator and it is on top of a division bar….. and we call 10 a denominator and it is underneath the division bar or fraction bar… because one (which is a numerator) is smaller than ten (which is a denominator), then we call this fraction a proper fraction…………………………………………………………………….(L13)*

When presenting fractions, Ms. Potsane started first by activating the learners' prior knowledge. The question that she asked (see L1) was motivated by her quest to first check if learners knew what the term sharing meant. To her, the *"sharing concept"* was important for learners to build on new information (fraction concepts) and to understand it in-depth. According to the UDL (neuroscience-based) framework, activating prior knowledge is critical for learners to internalise and assimilate new information.[46] Teaching that prioritises the activation of prior knowledge makes content accessible and easily understood (Sedumedi, 2008). Since some learners may have prior knowledge but may not even realise that it is pertinent to build on new information and knowledge, UDL thus requires the teachers to activate it and to show how it links with new information.[47] Ms. Potsane's choice of example, "*sharing"* served as one way of recruiting the learners' attention and interest. Sharing is a term that is not only used in class but also used outside the classroom. Therefore, learners could relate to its application in class, since it is the same as when applied in real life. The use of real-life examples in mathematics classrooms is critical to show the applicability of the concepts learnt in class in real life, thus optimising relevance, value and authenticity.[48] The practice enables learners to recognise the relevance of the concepts they are learning in class and to realise their applicability outside the classroom. Such practice is not only essential to make information accessible but to teach learners how to transform accessible information into usable knowledge.

---

[46] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[47] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[48] Pauline Vos, "'How Real People Really Need Mathematics in the Real World'—Authenticity in Mathematics Education," *Education Sciences* 8, no. 4 (2018): 195; CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.

To make the fractions accessible and comprehensible, Ms. Potsane used a cake as an example (representing a whole). The learners already know what a cake is and are familiar with the different cuts that are performed to allow for sharing to take place. Using such a "practical" example in her class made the concept simple, intuitive, and perceptible.[49] UDL espouses this type of practice as it eliminates threats and distractions that might hinder the learning process.[50]

Ms. Potsane managed to represent the concept in various formats (textually "one out of ten", numerically "1/10" and pictorially see Figure 1 above). According to UDL, the neuroscience-based framework customises content display by providing choices in the form of visual information, thus heightening perception and reinforcing understanding.[51] In her teaching, using the same example (cake), Ms. Potsane was further able to explicitly teach the vocabulary related to the "fractions" concept (see L13). Research shows that understanding mathematical vocabulary improves problem-solving and reinforces conceptual understanding of mathematical concepts.[52] After all, how can the learners participate, solve problems, and give answers when they do not understand the questions and what is expected of them?

The use of multiple means of representation through customising information display, teaching explicit vocabulary and eliciting prior knowledge, activates the recognition brain network, which makes it possible for learners to comprehend the content (fractions). The use of real-life objects to represent content in a different form, not only activates the recognition network, but also activates the affective network which assists in recruiting learners' attention, interest, and motivation to learn. Furthermore, the use of multiple means of representation as a varied method activates the strategic brain network which subsequently assists learners to develop strategies for solving problems and providing solutions.

Checking Learner understanding of Fractions when represented in various forms

Checking learner understanding during teaching is important to ensure that they retain the information while they are being taught. To check learner understanding, Ms. Potsane projected several models of fraction representations. She then asked learners which fraction these models represented.

Lesson description: demonstrating various representations of a fraction

Ms. Potsane: *...I want you to look at the following pictures and tell me which fraction is represented?………………………………………………………../.(L14)*

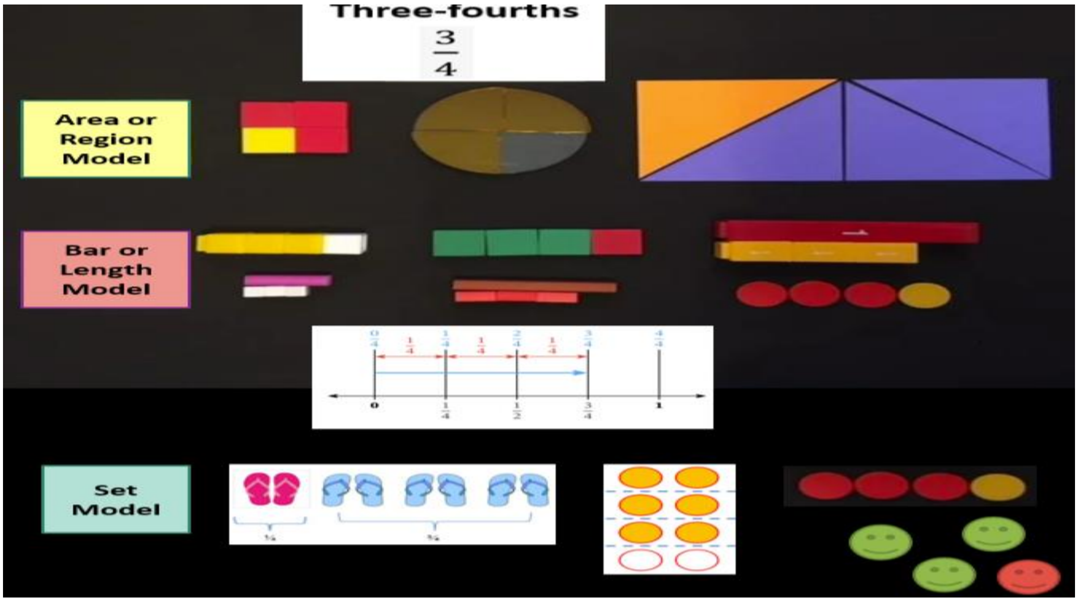


Figure 2: Different models of representations

[49] Sheryl Burgstahler, "Universal Design of Instruction (UDI): Definition," *Principles, Guidelines, and Examples*, 2009.
[50] Sandra Levey, "Universal Design for Learning," *Journal of Education* 203, no. 2 (2023): 479–87.
[51] Matshidiso M Moleko and Mogege D Mosimege, "Flexible Teaching of Mathematics Word Problems through Multiple Means of Representation," *Pythagoras-Journal of the Association for Mathematics Education of South Africa* 42, no. 1 (2021): 575.
[52] Benson Wanjiru, "Effects of Mathematical Vocabulary Instruction on Students' Achievement in Mathematics in Secondary Schools of Murang'a County, Kenya.," *Journal of Education and Practice* 6, no. 18 (2015): 201–7.

After showing learners how 1/10 can be represented using a cake example, Ms. Potsane decided to illustrate other forms of representations that are used when teaching fractions. Later in the lesson, she gave learners pictorial representations and asked them to give the numerical fraction that they represented. She showed learners various representations because she wanted to diversify the repertoire of representations used when dealing with fractions, with the belief that once they (learners), were aware of these representations, it would be easier for them to deal with any type (representation) of a fraction given to them. The teacher thus used these representations as customised-embedded scaffolds to build a strong, solid foundation to reinforce the understanding of fractions.

Figure 2 shows the different forms of representations which Ms. Potsane used to demonstrate other ways in which the numeric fraction, ¾ can be represented. The UDL framework endorses this form of practice to develop fluencies with regulated levels of support for practice and performance.[53] The practice further improves visualisation, which Rösken and Rolka regard as a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationships between them.[54] Representing content in various formats allows flexibility that is necessary to increase perceptual clarity and the salience of information for a diverse learner population, as well as alterations to the preferences of others. When multiple representations are used, the recognition brain network is activated and thus enables learners to recognise and comprehend the content. [55]

Varying the approaches for navigation, discovery and response

One of the challenges that learners experience when dealing with fractions is that they are unable to order them either in descending or ascending order. For instance, to some learners, 1/6 is bigger than ½ and therefore when ordering these fractions in an ascending manner, they start with ½ followed by 1/6 instead of 1/6 followed by ½. To address this mistake, Mrs. Phiri used multiple representations (Figure 3). All names are pseudonyms.

Lesson presentation extract:

Mrs. Phiri: *... Place the following fractions in a descending order* $\frac{1}{3}$ *; 1;* $\frac{1}{6}$ *&* $\frac{1}{2}$*………(L15)*

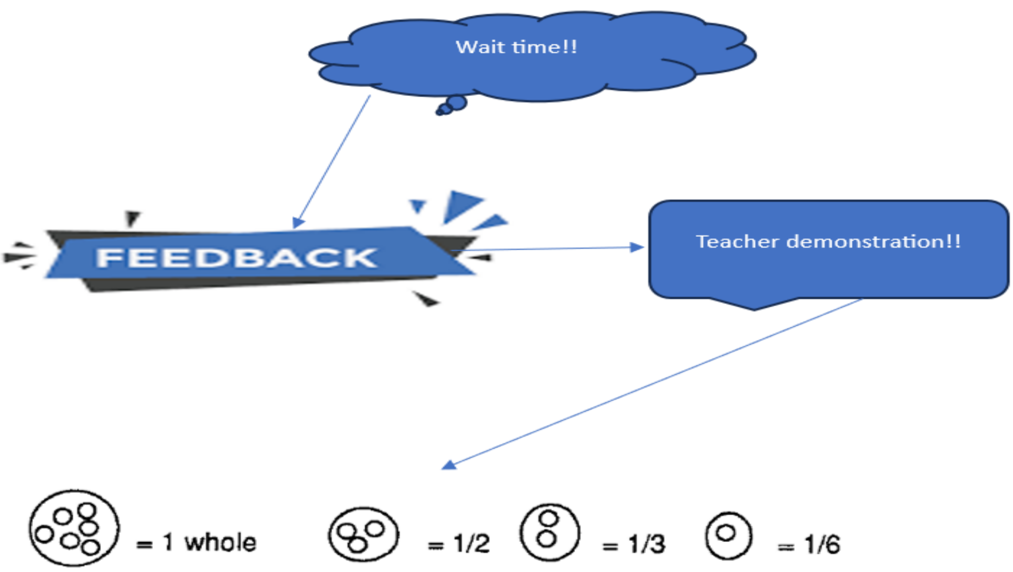


Figure 3: Ordering fractions in a descending order

[53] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.

[54] Bettina Rösken and Katrin Rolka, "A Picture Is Worth a 1000 Words–the Role of Visualization in Mathematics Learning," in *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, vol. 4 (Charles University Prague, 2006), 457–64.

[55] Alvyra Galkiene and Ona Monkeviciene, *Improving Inclusive Education through Universal Design for Learning* (Springer Nature, 2021).

Alma *(from group 5, shouting the answer!): 1/6; 1/3; ½; 1…………………(L16)*

Disebo *(from group 1): We also got the same answer Ma'm…………………(L17)*

Thabang *(from group 3): We got 1; ½; 1/3; 1/6………………………………(L18)*

Mrs. Phiri: *Tell me, what do you notice when the denominator increases? (referring to Figure 3)……………………………………………………………(L19)*

Thembi: *The value of the fraction becomes small…………………………..(L20)*

Nthabiseng: *Oh, I see why one over six is smaller than one. The circle is getting smaller and smaller …………………………………………………………………………………(L21)*

One of the most challenging and confusing exercises for many learners when dealing with fractions is that they find it difficult to order the fractions in either ascending or descending order. This confusion often arises due to a misconception that many learners have about the denominator. These learners tend to think that the bigger the denominator, the bigger the fraction; L16 and L17 indicate this misconception. Since Mrs. Phiri was aware of this misconception, she attempted to address it in her teaching by engaging her learners in an activity (i.e. ordering of the fractions in descending order), wherein they had to discover and decide on the ordering of these fractions from the biggest to the smallest.

When asking learners to provide the answer, she realised that most learners were indeed struggling to order the fractions [see L16 and L17]. In fact, they validated her, since she always believed that learners fail to order the fractions correctly because of the misconception they have regarding the denominator (i.e. the bigger the denominator, the bigger the fraction).

To address this misconception, Mrs. Phiri performed a demonstration using a visual representation (see Figure 3). The visual representation was intended to activate the brain recognition network to make the concept accessible and comprehensible. The visual representation made it easy for learners to recognise why the fraction 1/6 was the smallest, even though its denominator was the biggest.

The visual demonstration (Figure 3) that Mrs. Phiri used in class served as a form of constructive feedback intended to address the learners' misconceptions. The feedback was relevant and timely. According to the UDL framework, this is the type of feedback that activates the affective brain network which is essential for fuelling learners to learn. Her demonstration served as mastery-oriented feedback, which motivated and helped to sustain learning. The teacher's feedback thus made it possible for learners to recognise the relationships and critical features of fractions. Hence, the responses from Thembi and Nthabiseng respectively: ("*The value of the fraction becomes small*; *"Oh, I see why one over six is smaller than one. The circle is getting smaller and smalle*r").

Providing equal access to content (equivalent fractions) through alternatives for visual information

Recognising and representing equivalent fractions can be problematic for some learners. The problem can be attributed to many reasons including a learner's inability to count. For instance, some learners cannot identify multiples when they are represented in the form of fractions, although they can easily recognise them when represented in table form. Since Mrs. Phiri was aware of this issue, she used multiple representations as an intervention strategy that she incorporated into her equivalent fraction teaching lesson to enable learners to recognise and understand the concept (equivalent fractions). The extract below indicates how her teaching unfolded. All names are pseudonyms.

Lesson presentation extract

Mrs. Phiri: *You will remember last time when we were dealing with equivalent fractions. By the way, what are equivalent fractions? What did we say they are?..(L22)*

Lerato: *Ma'm, equivalent fractions are fractions that have different numerators and denominators but have the same value……………………………………(L23)*

Mrs Phiri: *Excellent! Now, can you give me two equivalent fractions of ½? Work in your groups and give me answers……………………………………………(L24)*

Tshepo: *(from group2): two over four………………………………………….(L25)*

Nandi: *(from group5): eight over sixteen……………………………………..(L26)*

Mrs. Phiri: *Well done!..(L27)*

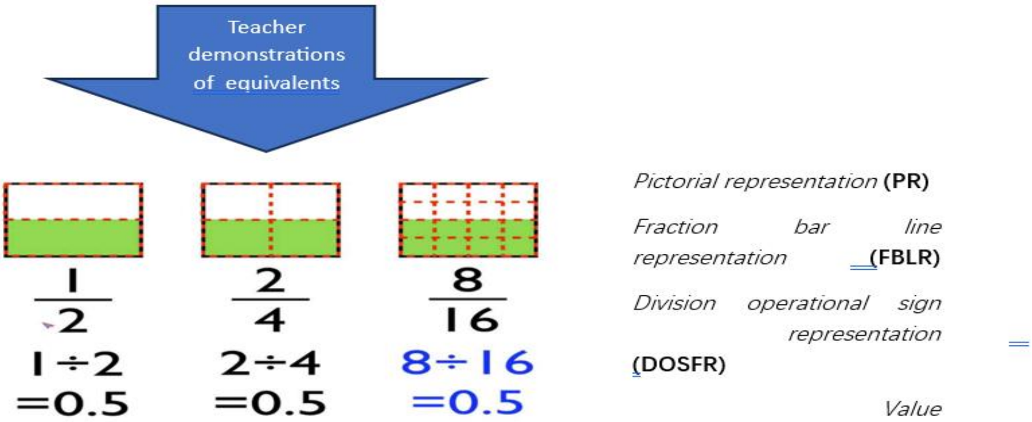


Figure 4: Equivalent fractions of different representations

When dealing with fractions, one of the challenges that most learners face is to recognise equivalent fractions (i.e. fractions that have different numerators and denominators but have the same value, which is the definition that Lerato provided - see L23). To ensure that learners do not simply recognise equivalent fractions, but also understand what makes them equivalent, Mrs. Phiri used multiple means of the representation principle. However, before using the principle, she saw fit to start by activating learners' prior knowledge in the same way as Ms. Potsane (see L1). Upon realising that some learners knew that equivalent fractions are based on the definitions they provided, she then gave them an activity to check whether they could represent the equivalent fractions numerically. This was to make sure that they did not just know the definition but could also demonstrate understanding by providing examples of such fractions in writing. To do the activity, she requested the learners to work collaboratively. According to the UDL framework, fostering collaboration and community is one of the important 21st-century skills which teachers must develop in learners.[56]

Although working in groups might be easier for some learners than others, it remains a goal for all learners. Research shows that this form of practice has the potential of increasing excitement for and engagement in the lesson and learning.[57] Furthermore, research shows that when the strategy is carefully structured, such peer collaboration can considerably increase the available support for sustained engagement.[58] Since Mrs. Phiri's goal was to ensure that learners understood what the equivalent fractions were and recognised them when they are presented, she deemed it fit to provide different forms of representations. According to Piaget's model of developmental phases, learners who are at the concrete operational stage (intermediate phase in the SA school context), tend to struggle with abstract and hypothetical concepts.[59] Thus, Mrs. Phiri began to demonstrate the concept of equivalent fractions by using a pictorial representation (see Figure 4 PR), followed by a fraction bar line representation (see Figure 4 FBLR) and division operational sign fraction representation (see Figure 4 DOSFR). Finally, she indicated the common value of the three fractions (equivalents). The use of the different representations thus clarified the definition of equivalent fractions that Lerato had provided (see

[56] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[57] Emily L Mofield, "Benefits and Barriers to Collaboration and Co-Teaching: Examining Perspectives of Gifted Education Teachers and General Education Teachers," *Gifted Child Today* 43, no. 1 (2020): 20–33.
[58] Moeketsi Freddie Tlali, "Transformational Learning of Physical Science through Service Learning for Sustainability" (University of the Free State, 2013).
[59] Rachana Maurya and Faziullah Khan, "Cognitive Development in Children with Autism Spectrum Disorder: A Piaget's Cognitive Developmental Approach," *Mind and Society* 10, no. 03–04 (2021): 117–24.

L23). The UDL practice (multiple representations) used by the teacher made the concept simple, intuitive and perceptible.[60] The practice made the concept explicit by clarifying the structural relationships between the elements (i.e. numerators and denominators) of the given equivalent fractions. Furthermore, the strategy ensures that all learners have equal access to information through alternatives to visual information.[61]

Sustaining learning through meaningful feedback

Teaching and learning cannot be separated from assessment. It is essential to provide learners with feedback based on what they have been assessed. The literature stresses the need for meaningful feedback to enhance the quality of learning.[62] Corroborating the same sentiment, during the focus group discussion Ms. Sefako said:

> *"…It helps to give them detailed feedback that shows them where they are wrong. It takes time but it is worth it. I gave them a fraction problem to solve and after identifying certain errors and misconceptions that they had, I saw the need to give them feedback that would help them recognise where they went wrong"……………………………………………………………………………………………….(L2 8)*

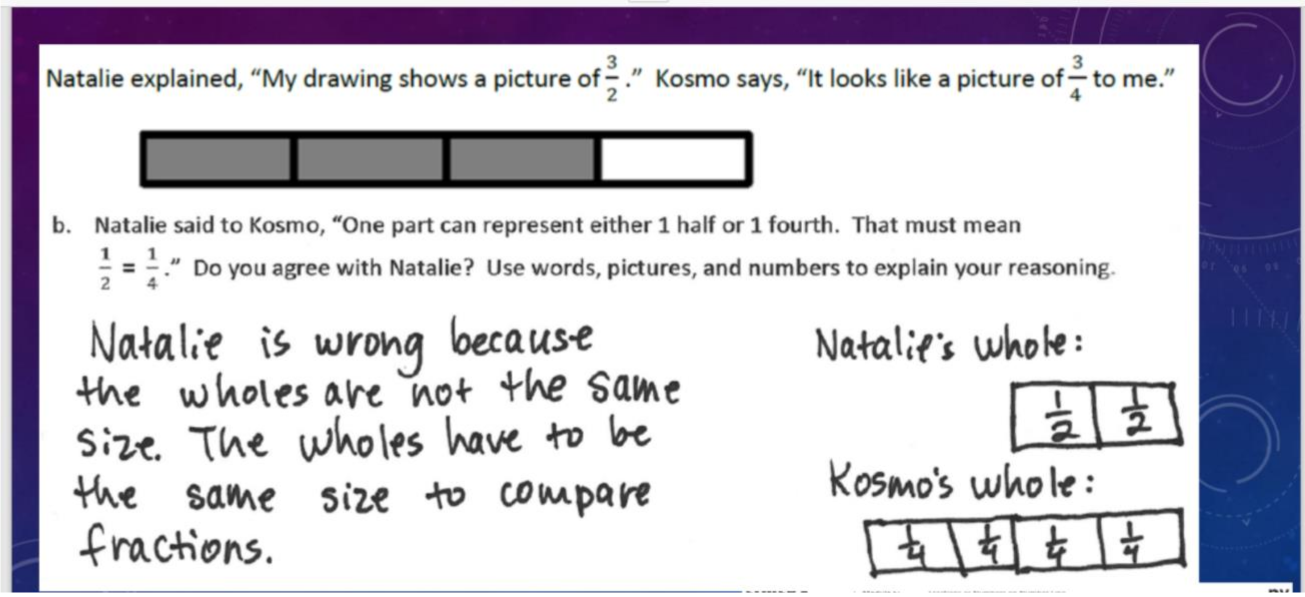


Figure 5: Differentiated feedback

According to Ms. Sefako, detailed feedback is crucial for sustaining and promoting meaningful learning. The statement *"It helps to give them detailed feedback that shows them where they are wrong"*, denotes that she uses feedback as a tool for guiding learners towards mastery of the content.[63] Although she acknowledges that such detailed feedback is time-consuming, she still believes it is worthwhile. In fact, she perceives this form of feedback as an opportunity for her to identify errors, which subsequently assist her in devising strategies that will address the wrong answers. This practice is in line with the UDL framework which requires teachers to provide feedback that models ways to integrate assessment, including the identification of mistakes into positive strategies for future success.[64] Looking at Figure 5, one realises that Ms. Sefako gave both textual and pictorial feedback. According to the UDL framework, this is called differentiated feedback, tailor-made to be accessible to individual learners.[65]

---

[60] Burgstahler, "Universal Design of Instruction (UDI): Definition."
[61] Dalton, "Beyond Universal Design for Learning: Guiding Principles to Reduce Barriers to Digital & Media Literacy Competence."
[62] Dai Hounsell, "Towards More Sustainable Feedback to Students," in *Rethinking Assessment in Higher Education* (Routledge, 2007), 111–23.
[63] Eunice Eunhee Jang and Maryam Wagner, "Diagnostic Feedback in the Classroom," *The Companion to Language Assessment* 2 (2013): 693–711.
[64] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[65] Joan M McGuire, Sally S Scott, and Stan F Shaw, "Universal Design for Instruction: The Paradigm, Its Principles, and Products for Enhancing Instructional Access.," *Journal of Postsecondary Education and Disability* 17, no. 1 (2003): 11–21.

Selecting the appropriate resources and differentiating activities

To ensure that all learners in the class make progress and desire to learn, teachers need to give tasks that learners can solve. Since learners have different strengths, is it important to differentiate these tasks. Of vital significance, is for teachers to choose the appropriate resources that will help learners comprehend the tasks. When highlighting these points, the participants commented as follows. All names are pseudonyms.

> ***Mr. Phophi:*** *When I teach my learners, I try to allocate to them activities that challenge them differently. I do this because I know that they have different strengths and different levels of motivation…………………………………(L29)*

> ***Ms. Sefako:*** *I also do the same. Furthermore, I make sure that I take time to choose the resources that I am going to use to teach my learners and also the ones that they are going to use to be able to understand and complete the tasks………………………………………………………………………………(L30)*

The extracts above allude to the significance of differentiating the activities according to the levels of demand. Mr. Phophi's utterance, "*I try to allocate to them activities that challenge them differently…"* signifies his efforts to vary the activities according to the different levels of demand and strength each learner has. This means that he allocates activities to the learners according to their different strengths and to the levels at which they can assimilate and comprehend the content. Those who struggle to grasp the concepts are given tasks/problems that they can solve. Similarly, those who do not struggle to grasp the concepts are also given tasks/problems that match their strengths. UDL endorses the practice of varying task demands to sustain effort and determination on the part of the learners.[66] Research shows that learners differ not only in their abilities and skills but also in the kinds of challenges that inspire them to perform well.[67] In line with this, therefore, the UDL framework requires that the learners be challenged but not in the same way.[68] The statement "*I do this because I know that they have different strengths and different levels of motivation…"* signifies the fact that Mr. Phophi knows his learners (i.e. their strengths and weaknesses). As a result, he manages to vary the tasks according to their capabilities and strengths. Differentiating activities assist in building fluencies and confidence, as well as enabling teachers to provide modified levels of support for practice and performance.

Ms. Sefako added that she takes time to select the resources that she uses to teach her learners. She also takes time to choose the resources that are going to be used by the learners to complete the tasks. Research shows that learners find it difficult to meet a demand without the appropriate, and flexible resources.[69] Thus, it is essential to provide a variety of demands and a variety of resources that will allow all learners to experience motivational challenges. The UDL framework endorses this practice as it provides learners with the right kinds of resources necessary to successfully complete the tasks.[70]

> ***Mr. Phophi:*** *Some learners understand quickly when you explain to them what one over four is. However, some find it difficult to understand the explanation. That is why I use the manipulatives and engage them so that they can understand and discover the answers on their own. It helps to give them clues…it helps them to think about the problem and respond.*

To assist learners who cannot understand the explanations which are given to them verbally in class, Mr. Phophi uses manipulatives. The statement, "*I use the manipulatives and engage them so that they can understand and discover the answers on their own…*" shows that the teacher uses the manipulatives to assist learners to explore and discover some concepts on their own. This form of practice is recommended by the UDL framework since it helps teachers vary the methods for response and navigation.[71] Research shows that learners differ extensively in their capacity to navigate their physical environment.[72] Therefore, to address barriers to learning that might be caused by the demands of a task, teachers need to provide learners with an alternative

---

[66] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[67] Monique Boekaerts, "Self-Regulated Learning: A New Concept Embraced by Researchers, Policy Makers, Educators, Teachers, and Students," *Learning and Instruction* 7, no. 2 (1997): 161–86.
[68] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[69] Ronghuai Huang et al., "Handbook on Facilitating Flexible Learning during Educational Disruption: The Chinese Experience in Maintaining Undisrupted Learning in COVID-19 Outbreak," *Beijing: Smart Learning Institute of Beijing Normal University* 46 (2020).
[70] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.
[71] Dalton, "Beyond Universal Design for Learning: Guiding Principles to Reduce Barriers to Digital & Media Literacy Competence."
[72] CAST, *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*, 2018.

means of response. Varying methods for navigation and response serve as a way in which teachers can provide learners with equal opportunities for interaction with learning experiences.

RECOMMENDATIONS

The research found that UDL (neuroscience-based teaching framework) can be used to make content (fractions) accessible. Furthermore, the research concluded that the framework could be used to cater for learner variability, scaffold learning, and the promotion of meaningful learning. Based on this therefore, it is recommended that the UDL guidelines be used to guide the planning and teaching of fractions in the intermediate phase. It is further recommended that workshops be conducted regularly to empower teachers with skills and knowledge to implement UDL in their classes.

CONCLUSION

This study presents an all-encompassing type of pedagogy which considers the concurrent use of representation, engagement, action and expression strategies for optimising and promoting meaningful learning. The study shows how the UDL (neuroscience-based) framework principles can be used to make content (fractions) accessible and comprehensible. Using UDL as a theoretical framework, the findings of the study revealed several practices that were applied to make content (fractions) accessible and comprehensible. The UDL framework exemplifies a neuroscience approach to teaching. UDL's multiple means of representation strategies, which activate the recognition brain network, aid learners to recognise and comprehend fractions.

The strategies included the use of different representations; activating prior knowledge; teaching explicit mathematical vocabulary; customising the display of concepts; making illustrations using multiple media and highlighting the relationships. Moreover, multiple means of engagement strategies afforded learners different opportunities for involvement. The strategies encompassed the use of multiple resources at a variety of levels to meet the unique needs of individual learners, thus reducing distractions and threats, offering 'mastery-oriented' feedback, and engaging and sustaining learners' interest and perseverance. These strategies activated the affective brain network which kept learners motivated and helped sustain learning. The multiple means of action and expression strategies used activated the strategic brain network which allowed learners to demonstrate their learning processes in various ways. These strategies included the use of diverse methods of demonstrating skills; varying approaches for navigating the problem and giving responses; building fluencies with progressive levels of support for practice and performance; sustaining learning through meaningful feedback; selecting the appropriate resources and differentiating the activities.

Inferring from the findings of the study for meaningful learning to take place, a neuroscience approach requires teachers to create a flexible, barrier-free learning environment where all learners have access to learning in a way that works for them. It requires teachers to use strategies that promote a shift from traditional methods to approaches that cater for a diverse learner population. It further requires teachers to value the inherent strengths and diversity of all learners, by designing proactive learning environments for variability. A neuroscience approach requires the three brain networks to be activated during the learning process, for learners to recognise and comprehend fractions, develop strategies to solve the fractions given to them and realise the applicability of the concepts they are learning in class in reality. The approach thus empowers the learners to be knowledgeable, strategic and motivated to learn.

Point of clarity

The study does not require the teachers to apply all the principles in one lesson. However, when a specific topic is presented, the UDL principles can be used concurrently or introduced after the other. It would still be appropriate to introduce one principle after the other since the study has also shown that these principles overlap and are intertwined.

BIBLIOGRAPHY

Barteaux, Susan. "Universal Design for Learning." *BU Journal of Graduate Studies in Education* 6, no. 1 (2014): 50–54.

Boekaerts, Monique. "Self-Regulated Learning: A New Concept Embraced by Researchers, Policy Makers, Educators, Teachers, and Students." *Learning and Instruction* 7, no. 2 (1997): 161–86.

Boggan, Matthew, Sallie Harper, and Anna Whitmire. "Using Manipulatives to Teach Elementary Mathematics." *Journal of Instructional Pedagogies* 3 (2010).

Booth, Julie L, and Kristie J Newton. "Fractions: Could They Really Be the Gatekeeper's Doorman?" *Contemporary Educational Psychology* 37, no. 4 (2012): 247–53.

Booth, Julie L, Kristie J Newton, and Laura K Twiss-Garrity. "The Impact of Fraction Magnitude Knowledge on Algebra Performance and Learning." *Journal of Experimental Child Psychology* 118 (2014): 110–18.

Boothe, Kathleen A, Marla J Lohmann, Kimberly A Donnell, and D Dean Hall. "Applying the Principles of Universal Design for Learning (UDL) in the College Classroom." *Journal of Special Education Apprenticeship* 7, no. 3 (2018): n3.

Bruer, John T. "Points of View: On the Implications of Neuroscience Research for Science Teaching and Learning: Are There Any? A Skeptical Theme and Variations: The Primacy of Psychology in the Science of Learning." *CBE—Life Sciences Education* 5, no. 2 (2006): 104–10.

Burgstahler, Sheryl. "Universal Design of Instruction (UDI): Definition." *Principles, Guidelines, and Examples*, 2009.

Carbonneau, Kira J, and Scott C Marley. "Instructional Guidance and Realism of Manipulatives Influence Preschool Children's Mathematics Learning." *The Journal of Experimental Education* 83, no. 4 (2015): 495–513.

CAST. *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*. Wakefield, MA: National Center on AEM, 2011.

———. *Universal Design for Learning Guidelines Version 2.0 [Graphic Organizer]*. Wakefield, MA: National Center on AEM, 2018.

Chamane, Cynthia Nonhlanhla. "Exploring Teachers' Experiences of Teaching Fractions in Grade 6 in the Curriculum and Assessment Policy Statement: A Case Study of One Rural School in Ndwedwe Circuit.," 2016.

Dalton, Elizabeth M. "Beyond Universal Design for Learning: Guiding Principles to Reduce Barriers to Digital & Media Literacy Competence." *Journal of Media Literacy Education* 9, no. 2 (2017): 17–29.

Drews, Doreen, and A Hansen. "Do Resources Matter in Primary Mathematics Teaching and Learning." *Using Resources to Support Mathematical Thinking, Primary and Early Years*, 2007, 19–31.

Empson, Susan B, and Linda Levi. "Extending Children's Mathematics: Fractions and Decimals." *Mathematics Education* 27, no. 4 (2011): 403–34.

Fazio, L. K., and R. S. Siegler. "Teaching Fractions." In *Educational Practices Series*. Geneva: International Academy of EducationInternational Bureau of Education, 2011.

Galkiene, Alvyra, and Ona Monkeviciene. *Improving Inclusive Education through Universal Design for Learning*. Springer Nature, 2021.

Getenet, Seyum, and Rosemary Callingham. "Teaching Interrelated Concepts of Fraction for Understanding and Teacher's Pedagogical Content Knowledge." *Mathematics Education Research Journal* 33 (2021): 201–21.

Gould, Peter, Lynne Outhred, and Michael Mitchelmore. "One-Third Is Three-Quarters of One-Half." In *Annual Meeting of the Mathematics Education Research Group of Australia, Australia*, 2006.

Guy, Richard, and Bruce Byrne. "Neuroscience and Learning: Implications for Teaching Practice." *Journal of Experimental Neuroscience* 7 (2013): 39.

Hackenberg, Amy J, and Mi Yeon Lee. "Relationships between Students' Fractional Knowledge and Equation Writing." *Journal for Research in Mathematics Education* 46, no. 2 (2015): 196–243.

Hounsell, Dai. "Towards More Sustainable Feedback to Students." In *Rethinking Assessment in Higher Education*, 111–23. Routledge, 2007.

Hoxha, M., and E. Vula. "Improving the Teaching of Fractions through Collaborative Research." *Albanian Journal of Educational Studies* 2, no. 2 (2014): 4–18.

Huang, Ronghuai, D J Liu, Ahmed Tlili, JunFeng Yang, and HuaHaun Wang. "Handbook on Facilitating Flexible Learning during Educational Disruption: The Chinese Experience in Maintaining Undisrupted Learning in COVID-19 Outbreak." *Beijing: Smart Learning Institute of Beijing Normal University* 46 (2020).

Jang, Eunice Eunhee, and Maryam Wagner. "Diagnostic Feedback in the Classroom." *The Companion to Language Assessment* 2 (2013): 693–711.

Jones, Julie P, and Margaret Tiller. "Using Concrete Manipulatives in Mathematical Instruction." *Dimensions of Early Childhood* 45, no. 1 (2017): 18–23.

Lazić, Bojan, Sergei Abramovich, Mirela Mrđa, and Daniel A Romano. "On the Teaching and Learning of Fractions through a Conceptual Generalization Approach." *International Electronic Journal of Mathematics Education* 12, no. 3 (2017): 749–67.

Lemonidis, Charalambos, and Ioanna Kaiafa. "The Effect of Using Storytelling Strategy on Students' Performance in Fractions." *Journal of Education and Learning* 8, no. 2 (2019): 165–75.

Levey, Sandra. "Universal Design for Learning." *Journal of Education* 203, no. 2 (2023): 479–87.

Lin, Cheng-Yao, Jerry Becker, Mi-Ran Byun, Der-Ching Yang, and Tsai-Wei Huang. "Preservice Teachers'

Conceptual and Procedural Knowledge of Fraction Operations: A Comparative Study of the U Nited S Tates and T Aiwan.” *School Science and Mathematics* 113, no. 1 (2013): 41–51.
Maurya, Rachana, and Faziullah Khan. “Cognitive Development in Children with Autism Spectrum Disorder: A Piaget’s Cognitive Developmental Approach.” *Mind and Society* 10, no. 03–04 (2021): 117–24.
McGuire, Joan M, Sally S Scott, and Stan F Shaw. “Universal Design for Instruction: The Paradigm, Its Principles, and Products for Enhancing Instructional Access.” *Journal of Postsecondary Education and Disability* 17, no. 1 (2003): 11–21.
Meulenberg-Buskens, I. “Free Attitude Interview Technique.” *London: Unpublished Notes*, 2011.
Mntunjani, Lindiwe M., Stanley A. Adendorff, and Sibawu W. Siyepu. “Foundation Phase Teachers’ Use of Manipulatives to Teach Number Concepts: A Critical Analysis.” *South African Journal of Childhood Education* 8, no. 1 (November 21, 2018). https://doi.org/10.4102/sajce.v8i1.495.
Mofield, Emily L. “Benefits and Barriers to Collaboration and Co-Teaching: Examining Perspectives of Gifted Education Teachers and General Education Teachers.” *Gifted Child Today* 43, no. 1 (2020): 20–33.
Möhring, Wenke, Nora S Newcombe, Susan C Levine, and Andrea Frick. “Spatial Proportional Reasoning Is Associated with Formal Knowledge about Fractions.” *Journal of Cognition and Development* 17, no. 1 (2016): 67–84.
Moleko, Matshidiso M. “Multiple Means of Engagement Strategies for Maximising the Learning of Mathematics in Pandemic-Regulated Classrooms.” *International Journal of Learning, Teaching and Educational Research* 20, no. 8 (2021): 68–90.
Moleko, Matshidiso M, and Mogege D Mosimege. “Flexible Teaching of Mathematics Word Problems through Multiple Means of Representation.” *Pythagoras-Journal of the Association for Mathematics Education of South Africa* 42, no. 1 (2021): 575.
Moleko, Mirriam Matshidiso, and Mogege D Mosimege. “Teachers’ and Learners’ Experiences for Guiding Effective Teaching and Learning of Mathematics Word Problems.” *Issues in Educational Research* 30, no. 4 (2020): 1375–94.
Moss, Joan, and Robbie Case. “Developing Children’s Understanding of the Rational Numbers: A New Model and an Experimental Curriculum.” *Journal for Research in Mathematics Education* 30, no. 2 (1999): 122–47.
Moyo, Methuseli, and France M Machaba. “Grade 9 Learners’ Understanding of Fraction Concepts: Equality of Fractions, Numerator and Denominator.” *Pythagoras* 42, no. 1 (2021): 602.
Mwakapenda, Willy. “Using Everyday Experiences in Teaching Mathematics: A Case Study of Change in Malawian Classrooms.” *Journal of the Southern African Association for Research in Mathematics, Science and Technology Education* 5, no. 1 (2001): 53–64.
Naiser, Emilie A, Wendy E Wright, and Robert M Capraro. “Teaching Fractions: Strategies Used for Teaching Fractions to Middle Grades Students.” *Journal of Research in Childhood Education* 18, no. 3 (2003): 193–98.
Ndalichako, Joyce Lazaro. “Analysis of Pupils’ Difficulties in Solving Questions Related to Fractions: The Case of Primary School Leaving Examination in Tanzania.” *Creative Education* 4, no. 09 (2013): 69.
Purwadi, I, I Sudiarta, and I Nengah Suparta. “The Effect of Concrete-Pictorial-Abstract Strategy toward Students’ Mathematical Conceptual Understanding and Mathematical Representation on Fractions.” *International Journal of Instruction* 12, no. 1 (2019): 1113–26.
Rao, Kavita, and Grace Meo. “Using Universal Design for Learning to Design Standards-Based Lessons.” *Sage Open* 6, no. 4 (2016): 2158244016680688.
Rösken, Bettina, and Katrin Rolka. “A Picture Is Worth a 1000 Words–the Role of Visualization in Mathematics Learning.” In *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, 4:457–64. Charles University Prague, 2006.
Siemon, Dianne, Kim Beswick, Kathy Brady, Julie Clark, Rhonda Faragher, and Elizabeth Warren. *Teaching Mathematics: Foundations to Middle Years*. University Of Tasmania, 2015.
Son, Ji-Won, and Sharon L Senk. “How Reform Curricula in the USA and Korea Present Multiplication and Division of Fractions.” *Educational Studies in Mathematics* 74 (2010): 117–42.
Tlali, Moeketsi Freddie. “Transformational Learning of Physical Science through Service Learning for Sustainability.” University of the Free State, 2013.
Torbeyns, Joke, Michael Schneider, Ziqiang Xin, and Robert S Siegler. “Bridging the Gap: Fraction Understanding Is Central to Mathematics Achievement in Students from Three Different Continents.” *Learning and Instruction* 37 (2015): 5–13.
Vitoria, Linda, Fauzi Fauzi, and Nadia Ananda. “Students’ Performance in Solving Problems Involving

Fractions." *Proceedings of AICS-Social Sciences* 7 (2017): 707–14.

Vos, Pauline. "'How Real People Really Need Mathematics in the Real World'—Authenticity in Mathematics Education." *Education Sciences* 8, no. 4 (2018): 195.

Wanjiru, Benson. "Effects of Mathematical Vocabulary Instruction on Students' Achievement in Mathematics in Secondary Schools of Murang'a County, Kenya." *Journal of Education and Practice* 6, no. 18 (2015): 201–7.

Wood, E Kay, and Pat Millichamp. "Changing the Learning Ethos in School." *Journal of In-Service Education* 26, no. 3 (2000): 499–515.

ABOUT AUTHORS

Matshidiso Mirriam Moleko is a Senior Lecturer at the University of South Africa. She is a member of AMESA, SAARMSTE and SAERA.

Percy Sepeng is a full Professor of Mathematics Education at the Department of Natural Sciences Teaching, Sol Plaatje University, Kimberley, South Africa. He holds a PhD from the Nelson Mandela University. He is a policy analyst and a member of both AMESA and SAARMSTE.