






Using hands-On Versus Virtual Manipulatives to Assist Basic Seven (7) Students in Understanding the Concept of Algebraic Expressions: A Case Study of Bagabaga Demonstration Junior High School

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ABSTRACT

This research study employs a quasi-experimental research design. The primary objective is to compare the effectiveness of hands-on and virtual manipulatives in enhancing the comprehension of the concept of algebraic expressions among Basic Seven (7) students at Bagabaga Demonstration Junior High School in Sagnarigu Municipality located in Tamale, Ghana. The research population comprised 100 students, selected through purposive and simple random sampling techniques. The population was divided into two groups: the Experimental group and the Control group. Prior to the intervention, both groups underwent a pretest, resulting in nearly identical mean and standard deviation scores (i.e., 1.08 and 1.44). Following the intervention, a Posttest was conducted to assess the impact of the intervention methods. The Experimental group achieved a mean score of 7.3, accompanied by a standard deviation of 5.9 while the Control group attained a mean score of 8.86, with a standard deviation of 1.5. These findings suggest that the use of hands-on and virtual manipulatives has a substantial influence on the understanding of the concept of algebraic expressions among the Bagabaga Demonstration Junior High School Basic Seven (7) students. The study recommends the use of diverse approaches which employ both hands-on and virtual manipulatives to address various learning styles, integrating them with other teaching strategies as well as continuous training of teachers to effectively integrate manipulatives into the classroom, and collaborate with colleagues to share best practices.

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INTRODUCTION

Meaningful educational activities and cognitive tools might improve students' active involvement in the teaching-learning process and encourage their reflections on the concepts and relations to be investigated. It is claimed that the usage of manipulatives not only increases students' conceptual understanding and problem-solving skills but also promotes their positive attitudes towards

mathematics since they supposedly provide “concrete experiences” that focus attention and increase motivation. A concrete experience in a mathematics context is defined not by its physical or real-world characteristics but rather by how meaningful connections it could make with other mathematical ideas and situations.¹

The use of effective teaching strategies is essential for advancing high standards in education and allowing a better comprehension of difficult ideas, particularly in areas like mathematics. Algebraic expression is one such difficult topic in mathematics that frequently poses difficulties for students due to its abstract nature. However, elementary school mathematics in the United States places less emphasis on the representation of relations and more emphasis on answer-oriented questions, which makes it difficult for elementary students to develop algebraic thinking.² Inadequacies were found in a study by Schliemann et.al. which included students' limited interpretations of equal signs, misperceptions about the meaning of letters and numbers, refusal to accept an expression, and difficulty solving equations with variables on both sides of the equal sign.³ According to the Chief Examiner's Reports on the 2022 Basic Education Certificate Examination, most of the candidates showed weakness in questions where the application of the knowledge of Algebraic expression was required. Additionally, research by Rosli, howed that students frequently make errors when generating two expressions, factoring, or simplifying algebraic fractures.⁴ Algebraic expressions are one of the mathematical concepts in which students frequently err, according to Fahmi and Marlina.⁵

This led to the quest to investigate novel strategies to improve students' learning experiences and their comprehension of mathematical ideas in the Sagnarigu Municipality of Ghana, where the Basic Seven (7) students at the Bagabaga Demonstration Junior High School have problems with the concept of Algebraic expressions. The existing perception is that traditional classroom instruction may not always provide hands-on experiences that are essential for establishing and solidifying comprehension. Thus, an investigation was created to compare the effectiveness of hands-on manipulatives and virtual manipulatives in assisting Basic Seven (7) students at Bagabaga Demonstration Junior High to overcome their difficulty in comprehending the concept of Algebraic expressions.

This study will compare the outcomes of the two approaches in terms of students' conceptual understanding, engagement, and overall academic performance (i.e., pre-test and post-test) to determine whether the use of hands-on manipulatives or virtual manipulatives would have a significant impact on students' understanding the concept of Algebraic expressions.

The research site was chosen to be Bagabaga Demonstration Junior High School because of its location within Sagnarigu Municipal and its affiliation as a demonstration school to Bagabaga College of Education. The study will explore the differences in students' comprehension, academic performance, and engagement when using these two different types of manipulatives, aiming to provide insights into the most effective approach for enhancing students' understanding of the concept of algebraic expressions at this educational level.

The findings of this study could offer significant insights into the efficacy of both hands-on and virtual manipulatives in enhancing students' comprehension of algebraic expressions. The study's findings could potentially aid schools within Sagnarigu Municipal, particularly those focusing on mathematics education, by informing educators, policymakers, and similar communities about the effectiveness of the use of manipulatives in teaching mathematical concepts to students at the basic level.

¹ Soner Durmus and Erol Karakirik, “Virtual Manipulatives in Mathematics Education: A Theoretical Framework.,” *Turkish Online Journal of Educational Technology-TOJET* 5, no. 1 (2006): 117–23.

² Jeremy Kilpatrick and Jane Swafford, *Helping Children Learn Mathematics* (National Academy Press, 2002).

³ Analúcia D. Schliemann et al., “Algebra in Elementary School,” *Proceedings of the 27th International Conference for the Psychology of Mathematics Education, 1–381* (Cape Town, South Africa: International Group for the Psychology of Mathematics Education, 2003).

⁴ D. Rosli, “Analisis Kesilapan Yang Dilakukan Oleh Pelajar Tingkatan Empat Dalam Menyelesaikan Masalah Berkaitan Ungkapan Algebra” (Tesis Sarjana Muda. Johor, Malaysia: Universiti Teknologi Malaysia, 2000).

⁵ Arzul Fahmi and Ali Marlina, “Analisis Kesilapan Dalam Tajuk Ungkapan Algebra Di Kalangan Pelajar Tingkatan Empat,” 2007.

Study Objectives:

1. How does the use of hands-on manipulatives in teaching Algebraic expressions affect students' comprehension and retention of the concept of Algebraic expressions?
2. How does the use of virtual manipulatives in teaching Algebraic expressions affect students' comprehension and retention of the concept of Algebraic expressions?
3. What are some of the benefits students' get from using manipulatives to learn a mathematical concept?
4. What are the perceptions and preferences of students regarding the use of hands-on manipulatives and virtual manipulatives in learning algebraic expressions?
5. To what extent do hands-on manipulatives and virtual manipulatives support students in developing a deeper conceptual understanding of Algebraic expressions?

LITERATURE REVIEW

Research and Benefits of the use of Manipulatives in Teaching

The use of manipulatives in education, particularly within the domains of mathematics and science, has been a subject of extensive research and deliberation over an extended period. These tangible resources, encompassing items like blocks, algebra tiles, counters, and models, have proven to be valuable instruments for enriching the educational process.

Researchers have found that the use of manipulatives in mathematics instruction can lead to improved student learning outcomes.⁶ Clements and Samara suggests that manipulatives have demonstrated their effectiveness in aiding students to cultivate a more profound conceptual grasp of scientific principles, notably in the fields of physics and chemistry.⁷ Studies suggest that manipulatives can improve students' problem-solving abilities by enabling them to investigate and test concrete materials.⁸ Added that manipulatives are known to increase student engagement and motivation, as they provide a hands-on and interactive approach to learning.⁹ Emphasized that manipulatives hold particular significance in early childhood education as they foster the development of fundamental math and literacy skills.

The Challenges Students Face When Learning the Concept of Algebraic Expression

Abstract Nature of Algebra

Algebraic expressions introduce abstract symbols (e.g., variables) that represent unknown quantities. Students may struggle with this shift from concrete arithmetic to abstract algebra. Kieran explained that a significant hurdle lies in the abstract character of algebraic expressions when students shift from dealing with tangible arithmetic to engaging with symbols and variables.¹⁰

Understanding Variables: Students may struggle with understanding the concept of variables and their role in representing unknown quantities.¹¹

Misconceptions and Errors: Misconceptions and errors in algebraic manipulation can persist if not addressed early, leading to difficulties in more advanced algebra topics. Students may find it challenging to understand the concept of variables and how they relate to real-world situations or problems.

⁶ E. J. Sowell, "The Impact of Manipulatives on Achievement in Mathematics: An Analysis of Eighth-Grade Data From the Longitudinal Study of American Youth," *Journal of Educational Research* 108, no. 4 (2015): 325–38.

⁷ Douglas H Clements and Julie Sarama, "Effects of a Preschool Mathematics Curriculum: Summative Research on the Building Blocks Project," *Journal for Research in Mathematics Education* 38, no. 2 (2007): 136–63.

⁸ K. Reimer, "The Effects of Manipulative Materials in Elementary Mathematics Instruction," *Action in Teacher Education* 32, no. 3 (2010): 33–44.

⁹ John A Van de Walle, Karen S Karp, and Jennifer M Bay-Williams, *Elementary and Middle School Mathematics* (Pearson, 2014).

¹⁰ Carolyn Kieran, "The Early Learning of Algebra: A Structural Perspective," in *Research Issues in the Learning and Teaching of Algebra* (Routledge, 2018), 33–56.

¹¹ John Mason, "Enabling Teachers to Be Real Teachers: Necessary Levels of Awareness and Structure of Attention," *Journal of Mathematics Teacher Education* 1, no. 3 (1998): 243–67.

Translating Word Problems: Translating word problems into algebraic expressions can be difficult, as students need to identify key information and variables to create equations. Students often find it challenging to translate word problems into algebraic expressions, particularly when identifying variables and setting up equations.¹²

Order of Operations: Weaknesses in basic arithmetic skills, such as multiplication, division, and order of operations, can hinder students' ability to simplify and manipulate algebraic expressions. Understanding and applying the correct order of operations (PEMDAS/BODMAS) can be problematic for students when simplifying algebraic expressions.¹³

Lack of Visualization: Some students struggle to visualize algebraic expressions and may benefit from visual aids or manipulatives to grasp abstract concepts.¹⁴

What is a Hands-on Manipulative

Hands-on manipulatives refer to physical objects or materials that students can physically handle, manipulate, and interact with during learning activities. These manipulatives are designed to provide a tangible representation of mathematical concepts, such as algebraic expressions, and allow students to explore and experiment with them concretely. Some examples of hands-on manipulatives for algebraic expressions are algebra tiles, counters, blocks, or other objects that can be manipulated and rearranged to represent different mathematical operations or relationships.

1. How does the use of hands-on manipulatives affect students' comprehension?

Students may find manipulative use exciting and motivating, which will inevitably increase their interest in the activity and the manipulatives' intended use. Children are naturally curious, playful and full of energy. According to research, algebraic expression skills are higher in students who used manipulatives in their mathematics classes than in those who did not.¹⁵ Beyond the lack of enjoyment, most students in a sit-and-listen mathematics lesson walk away with a low degree of understanding and retention.¹⁶

Jones claimed that, to teach young children mathematics, creativity and enthusiasm are just as effective as state-of-the-art equipment.¹⁷ Children's minds can be engaged by physical teaching tools for mathematics in meaningful ways that increase learning retention. However, using concrete manipulatives in math instruction can encourage student interest in mathematics.¹⁸ The use of hands-on interaction with concrete manipulatives allows students of all mathematical levels to begin instruction on a level playing field rather than diving right into an abstract concept.¹⁹

2. What is a Virtual Manipulative?

A virtual manipulative refers to an interactive digital tool or program that provides hands-on, interactive, and visual representation of mathematical concepts or models. It is aimed at enhancing the learning and understanding of mathematical concepts by allowing students to experiment, explore, and manipulate virtual objects or tools on a computer or other digital device. Virtual manipulatives

¹² Mary Hegarty and Kintsch Walter, "How Graphs Can Aid the Solution of Algebra Word Problems," *Instructional Science* 21, no. 1 (1992): 15–44.

¹³ Mitchell J. Nathan, Kenneth R. Koedinger, and Martha W. Alibali, "'I Only Count the Shot That I Think You're Gonna Take': Analysis of the Algebraic Activity of a Student and a Teacher in a Computer-Rich Classroom," *Cognition and Instruction* 19, no. 4 (2001): 329–73.

¹⁴ Kieran, "The Early Learning of Algebra: A Structural Perspective."

¹⁵ Michael F Chappell and Marilyn E Strutchens, "Creating Connections: Promoting Algebraic Thinking with Concrete Models," *Mathematics Teaching in the Middle School* 7, no. 1 (2001): 20–25.

¹⁶ Susuoroka Gabina, "The Effects of Using Manipulatives in Teaching and Learning of Algebraic Expression on Senior High School (SHS) One Students' Achievements in Wa Municipality," *Journal of Educational Development and Practice* 3, no. 3 (2019): 83–106.

¹⁷ Julie P Jones and Margaret Tiller, "Using Concrete Manipulatives in Mathematical Instruction.," *Dimensions of Early Childhood* 45, no. 1 (2017): 18–23.

¹⁸ Patricia Moch, "Manipulatives Work!," *The Educational Forum* 66, no. 1 (2001): 81–87.

¹⁹ Jones and Tiller, "Using Concrete Manipulatives in Mathematical Instruction."

commonly include virtual blocks, cubes, numbers, mathematical shapes, graphs, rulers, protractors, and other tools that students can use to solve problems, explore patterns, and manipulate objects in a virtual environment. A virtual manipulative is described as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge."²⁰

3. How does the use of virtual manipulatives affect students' comprehension?

According to the NCTM Technology Principle, "Work with virtual manipulatives, can allow young children to extend physical experience and to develop an initial understanding of sophisticated ideas like the use of algorithms." The National Council of Teachers of Mathematics suggests incorporating technology and mathematical tools as vital resources to address Mathematics Practice (MP) Standards, by using the right tools strategically.²¹

Virtual manipulatives are a new class of manipulatives that are generating new capabilities and toolkits for computer programs that use visual representations.²² Static and dynamic visual representations of concrete manipulatives are examples of virtual manipulatives.²³ Static visual representations include things like illustrations in books, drawings projected onto an overhead screen, and sketches on a chalkboard.

Virtual manipulatives, which were used in this study, are dynamic visual representations of concrete manipulatives that include computer-generated images and can be used in the same ways as concrete manipulatives. The dynamic virtual manipulative is a genuine virtual manipulative; however, the term is frequently used to describe any computer-generated image on a monitor that mimics concrete manipulatives.

4. What Benefits do students' get from using manipulatives to learn mathematical concepts?

Algebraic abilities include the ability to represent algebraic expressions, to interpret such expressions, to make connections between concepts when solving linear equations, and to communicate algebraic concepts. Research indicates that students who used manipulatives in their mathematics classes have higher algebraic abilities than those who did not use manipulatives.²⁴

A well-known math educator, Marilyn Burns, considers manipulatives essential for teaching math to students of all levels. She finds that manipulatives help make math concepts accessible to almost all learners, while at the same time offering ample opportunities to challenge students who catch on quickly to the concepts being taught. Research indicates that using manipulatives is especially useful for teaching low achievers and students with learning disabilities.²⁵

Research also indicates that using manipulatives helps improve the environment in math classrooms. When students work with manipulatives and then are given a chance to reflect on their experiences, not only is mathematical learning enhanced, but math anxiety is greatly reduced.²⁶ Exploring manipulatives, especially self-directed exploration, provides an exciting classroom environment and promotes in students a positive attitude toward learning. Among the benefits, several researchers found for using manipulatives was that they helped make learning fun.²⁷

In summary research from both learning theory and classroom studies shows that using manipulatives to help teach math can positively affect student learning. This is true for students at all

²⁰ Patricia S Moyer, Johnna J Bolyard, and Mark A Spikell, "Virtual Manipulatives in the K-12 Classroom,," 2001.

²¹ National Council of Teachers of Mathematics (NCTM), *Principles to Action: Ensuring Mathematical Success for All* (Reston, VA: National Council of Teachers of Mathematics, 2014).

²² Patricia S Moyer, "Are We Having Fun yet? How Teachers Use Manipulatives to Teach Mathematics," *Educational Studies in Mathematics* 47, no. 2 (2001): 175–97.

²³ J Spicer, "Virtual Manipulatives: A New Tool for Hands-on Math," *ENC Focus* 7, no. 4 (2000): 14–15.

²⁴ Chappell and Strutchens, "Creating Connections: Promoting Algebraic Thinking with Concrete Models."

²⁵ Lynn G Marsh and Nancy L Cooke, "The Effects of Using Manipulatives in Teaching Math Problem Solving to Students with Learning Disabilities,," *Learning Disabilities Research and Practice* 11, no. 1 (1996): 58–65.

²⁶ Marlene Cain-Caston, "Manipulative Queen," *Journal of Instructional Psychology* 23, no. 4 (1996): 270.

²⁷ Moch, "Manipulatives Work!"

levels and of all abilities. It is also true for almost every topic covered in elementary school mathematics curricula.

Papert calls manipulatives “objects to think with.”²⁸ Incorporating manipulatives into mathematics lessons in meaningful ways helps students grasp concepts with greater ease, making teaching the most effective.

METHODOLOGY

Research Design

The research design adopted for this study was the quasi-experimental research method. This design was adopted because the study sought to compare the effectiveness of using hands-on manipulatives against virtual manipulatives in teaching and learning the concept of Algebraic expression in Bagabaga Demonstration Junior High School in the Sagnarigu Municipality.

The control group (C) was exposed to virtual manipulatives while the experimental group (E) used hands-on manipulatives. The two groups were afterwards compared to evaluate the effectiveness of each method in helping Basic seven (7) students of Bagabaga Demonstration Junior High School to overcome their difficulty in understanding the concept of algebraic expressions.

Population

The participants of this study comprised all the Basic Seven (7) students from Bagabaga Demonstration Junior High School. A total of 100 students from two intact classes were purposively selected for this research, with one class assigned as the control group (C) and the other as the experimental group (E).

Sample Procedure

A purposive sampling technique was used to select the classes for the study. The decision was made based on the classes' prior performance and willingness to participate in the research. To minimize bias, a random assignment of groups was conducted using a simple random sampling technique. A box containing 100 well-folded pieces was presented to the students. 50 pieces had E written on them denoting the experimental group and the other 50 pieces had C also written on them denoting the control group. The students were asked to randomly select a piece of paper from the box to indicate the group they would belong to, this meant at the end of the exercise each group had 50 well-integrated participants (i.e., 50 for the experimental group and 50 for the control group).

Data Collection Instruments

Pretest

Before the intervention, both groups took a pretest to assess their initial understanding of algebraic expressions. This will help tailor the intervention to address the specific needs of the students effectively.

The pretest consisted of 10 questions related to the concepts of algebraic expressions. Below are the questions:

1. What is an algebraic expression?
2. How do you identify the terms in an algebraic expression?
3. Explain the difference between a coefficient and a constant in an algebraic expression.
4. How can you simplify an algebraic expression by combining like terms?
5. What does it mean to evaluate an algebraic expression?
6. How do you determine the value of a specific variable in an algebraic expression?
7. Explain the meaning of the distributive property in relation to algebraic expressions.
8. Give an example of an algebraic expression that represents “twice a number increased by 5.”
9. How can you use an algebraic expression to solve real-life problems or mathematical equations?

²⁸ Seymour Papert, *Mindstorms* (Scranton, PA: Basic Books, 1980).

10. Describe the steps to simplify the expression $3(x + 2) - 5x$.

Intervention Phase

Both the control and experimental groups attended separate instructional sessions using their respective manipulatives. The duration of the intervention spanned 5 weeks to allow the students sufficient exposure to the manipulatives and the intended concepts.

Development of Algebraic Expression Conceptual Knowledge (to be considered later)

Developing conceptual knowledge of algebraic expressions involves understanding the fundamental concepts that underlie these mathematical expressions. Here is a step-by-step guide to help individuals build a solid conceptual foundation in algebraic expressions:

Step 1: Understanding Arithmetic Operations

- Begin with a strong grasp of basic arithmetic operations, including addition, subtraction, multiplication, and division.
- Ensure a solid understanding of the relationship between numbers and the outcomes of these operations.

Step 2: Introduction to Variables

- Introduce the concept of variables as symbols that represent unknown or changing quantities.
- Explain that variables are often denoted by letters such as 'x' or 'y.'

Step 3: Recognizing Constants and Variables

- Teach individuals to distinguish between constants (fixed numbers) and variables (symbols that represent changing values).
- Show examples like " $3x$," where "3" is a constant, and "x" is a variable.

Step 4: Building Simple Expressions

- Show how to create basic algebraic expressions by combining constants and variables with arithmetic operations.
- Examples include " $2x + 3$ " or " $5y - 1$."

Step 5: Understanding Like Terms

- Explain the concept of like terms, which are terms that have the same variables and exponents.
- Emphasize the importance of like terms for simplifying expressions.

Step 6: Simplifying Expressions

- Teach how to simplify algebraic expressions by combining like terms using addition and subtraction.
- Practice simplifying expressions like " $2x + 3x$ " into " $5x$."

Step 7: Introduction to Equations

- Introduce equations as mathematical statements that show equality between two algebraic expressions.
- Show simple equations like " $2x = 10$ " and explain that the goal is to find the value of 'x.'

Step 8: Solving Equations

- Teach techniques for solving equations, such as isolating the variable by performing inverse operations.
- Solve equations like " $3x - 6 = 12$ " to find the value of 'x.'

Step 9: Translating Word Problems

- Practice translating word problems into algebraic expressions and equations.
- Show how to identify keywords that indicate addition, subtraction, multiplication, or division in word problems.

Step 10: Exploring More Complex Expressions

- Gradually introduce more complex algebraic expressions with multiple variables, exponents, and parentheses.
- Examples include " $2x^2 - 3xy + 5$ " or " $3(x + 2) - 4$."

Step 11: Factoring and Expanding

- Teach factoring to break down algebraic expressions into simpler forms, and expanding to reverse the process.
- Show examples like factoring " $x^2 - 4$ " into " $(x + 2)(x - 2)$ " and expanding " $(2x + 3)(x - 1)$."

By following these steps and gradually progressing from fundamental concepts to more advanced topics, individuals can develop a strong conceptual understanding of algebraic expressions. This conceptual foundation is essential for mastering algebra and its applications in mathematics and beyond.

Intervention Activities

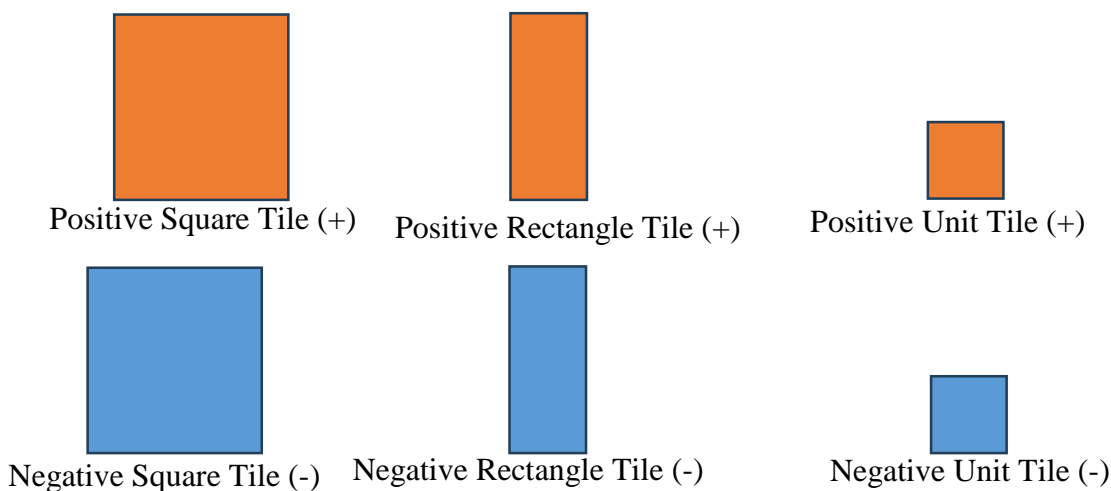
Both the C and E groups received instruction on algebraic expressions using their respective manipulative tools. The C group used virtual manipulatives on computers or tablets while the E group used hands-on manipulatives like blocks or algebra tiles. The instruction provided was standardized across both groups and delivered by the same teachers to ensure consistency.

Week 1 Activities

The following activities took place in the first week of the intervention for both groups.

Introduction to the Algebra Tiles

The class was started by acquainting students with algebra tiles (i.e., positive is brown tiles and negative is blue tiles), tangible tools specifically designed to illustrate algebraic expressions visually. The students were divided into ten groups, each comprising five students, and provided ample time for hands-on interaction to become acquainted with the algebra tiles.




Representation of the Algebraic Tiles


In the different groups, the function of each algebra tile in constructing algebraic expressions were explained to the students.


i. Small square tiles represent unit constants (e.g., "1").

For example,


Represents the constant 1
in Algebraic expression



Represent the constant 2
in Algebraic expression



Represents the constant -1
in Algebraic expression



Represents the constant -2
in Algebraic expression

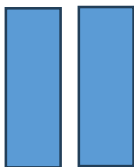
ii. Rectangle tiles represents the variable "x".

For example,


Represents the positive variable x
in Algebraic expression



Represent the positive variable 2x
in Algebraic expression



Represents the negative variables -x
in Algebraic expression



Represents the negative variables -2x
in Algebraic expression


iii. Large square tiles represent squared the variable "x²".

For example,


Represents the variable x²
in Algebraic expression

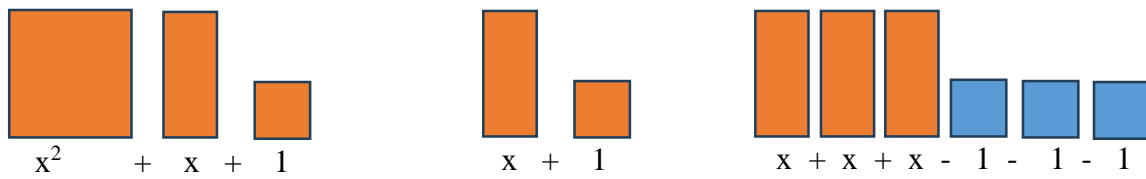

Represent the variable 2x²
in Algebraic expression


Represents the variable -x²
in Algebraic expression


Represent the variable -2x²
in Algebraic expression

Modeling a Simple Algebraic Expressions

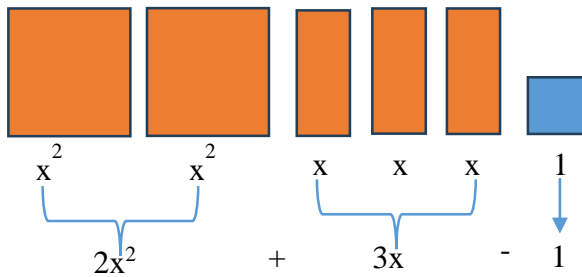
Once they had become familiar with the fundamental algebra tiles, the students in both E and C groups were directed to illustrate elementary algebraic expressions like " $x^2 + x + 1$," " $x + 2$," or " $3x - 3$ " using algebra tiles. They demonstrated how to visually represent these expressions using the corresponding algebra tiles as follows.



Week 2 Activities

Combining Tiles

Students in both E and C groups were taught how to combine tiles to represent more complex expressions, like " $2x^2 + 3x - 1$." The importance of arranging tiles to match the structure of the expression was emphasized. Every group had complex expressions to arrange as follows.



Addition and Subtraction of Algebraic Expressions

The researchers guided the students in C and E groups in illustrating the addition and subtraction of algebraic expressions using algebra tiles. As an illustration, consider solving the following.

- i. $(x + 3) + (2x + 1)$
- ii. $(x^2 + x + 2) + (x^2 + 2x + 1)$
- iii. $(5x + 3) - (2x + 1)$
- iv. $(2x^2 + x + 2) - (x^2 + 2x + 2)$

Solution:

i.

$$x + 1 + 1 + 1 + x + x + 1 = 3x + 4$$

ii.

$$x^2 + x + 2x^2 + 2x + 1 = 2x^2 + 3x + 3$$

iii.

$$5x + (-2x - 1) = 3x +$$

iv.

$$x^2 + x^2 + x + 1 + 1 + (-x^2 - x - x - 1 - 1) = x^2 -$$

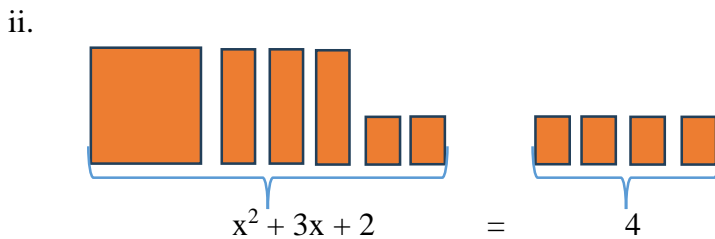
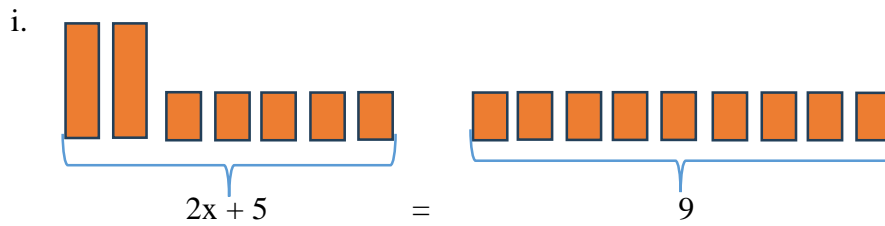
Visualization of Equations

Question: The researchers offered instructions to students in the E and C groups, teaching them how to illustrate equation modeling by ensuring that the algebra tiles on either side of the algebraic expressions were in equilibrium.

To illustrate, let's examine the following algebraic equations.

i. $2x + 5 = 9$

ii. $x^2 + 3x + 2 = -4$



Week 3 Activities

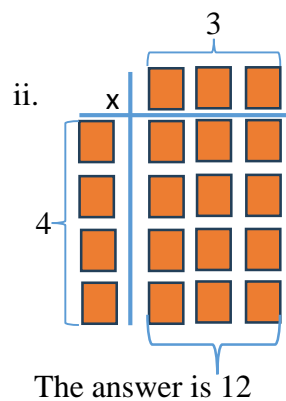
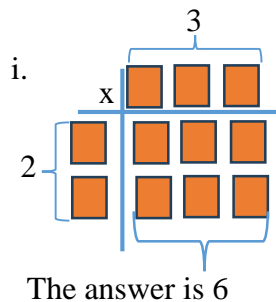
Multiplication with Algebra Tiles

In this exercise, individual groups in both E and C groups will employ the rectangular array method to represent the result of multiplying two polynomials. Position the initial factor vertically below the multiplication symbol and position the second factor horizontally to the right of the multiplication symbol and the numerical values of the concern tiles will be used in the multiplication processes. The numeric value of the tiles, symbolizing the first factor, is utilized to perform multiplication with the numeric value of the tiles placed horizontally to the right of the multiplication symbol, representing the second factor. The outcome or product corresponds to the summation of the tiles arranged in a rectangular fashion, positioned to the right of the initial factor and aligned horizontally beneath the second factor.

Activity 1: Unit by Unit

Model the following multiplication:

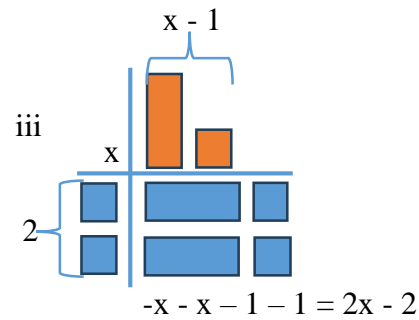
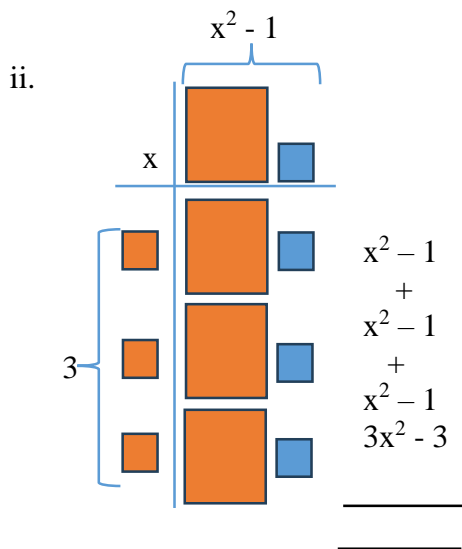
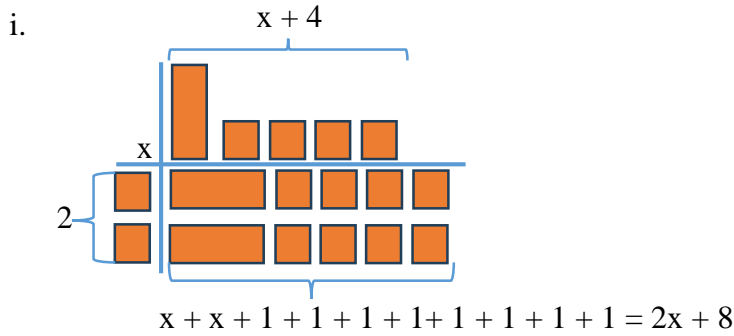
- i. 2×3
- ii. 4×3



Activity 2: Unit by Polynomial

Model the following multiplication:

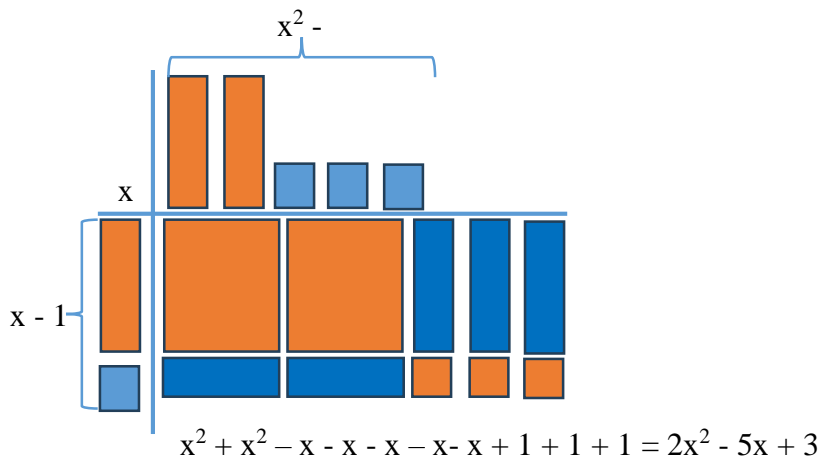
- i. $2(x + 4)$
- ii. $3(x^2 - 1)$
- iii. $-2(x + 1)$



Activity 3: Polynomial by Polynomial

Model the following multiplication

$(x - 1)(2x - 3)$



Week 4 Activities

Activity 1: Algebraic Manipulations

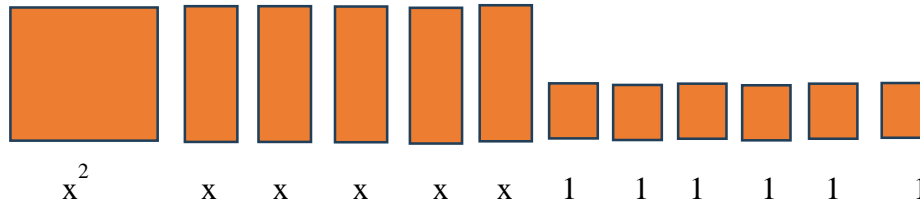
This phase marked the application stage, during which students in the E and C groups were tasked with using their acquired knowledge to engage in a range of algebraic manipulations, such as factorization and expansion, through the use of algebraic tiles. Show how these manipulations can be

visualized with the tiles. The various groups were assigned distinct yet equivalent algebraic expression challenges to address.

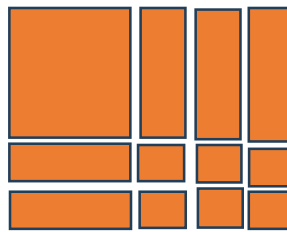
For example, factorize $x^2 + 5x + 6$

How to go about it: Method

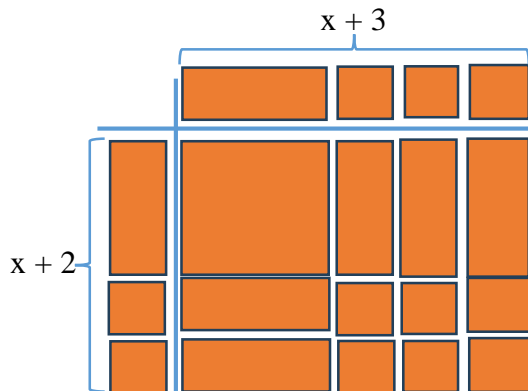
(i) Choose the tiles that symbolize the product, which is the area.



(ii) Form a rectangular arrangement of these tiles by positioning large square tiles in the upper left corner and unit (1) tiles in the lower right corner.



(iii) Determine the dimensions, which are the factors, of the resulting rectangle.



We can see that, the product of the factors: $(x + 2)(x + 3) = x^2 + 5x + 6$

After students have gained proficiency in using algebra tiles for the purpose of factoring and have grasped the concept of factoring, elucidate the algebraic patterns of factoring. This can be accomplished through conventional methods, such as reversing the expansion process, breaking down the middle term, or employing various other algebraic techniques. Thus, $x^2 + 5x + 6 \Rightarrow x^2 + 2x + 3x + 6$

$$(x^2 + 2x) + (3x + 6)$$

$$\text{By factorizing: } x(x + 2) + 3(x + 2)$$


$$= (x + 2)(x + 3)$$

Activity 2: Solving Linear Equations


The purpose of this task was to provide students in E and C groups with a direct and hands-on encounter in solving linear equations by using tangible and virtual materials to represent abstract concepts. This involves physically manipulating objects through actions like pushing, pulling, and touching. As the saying goes, "I hear and I forget, I see and I remember, but I do and I grasp the concept."

Algebra tiles can be employed for resolving linear equations, offering an effective approach to demonstrate the concept of the balancing method. It helps students develop an intuitive sense of the principle of "performing identical operations on both sides of an equation" and the rationale behind it.


The assorted groups were assigned distinct yet equivalent linear algebraic expression equation challenges to address. Demonstrate the step-by-step approach to isolating the variable. For example, i) $x - 2 = 1$ ii) $x + 2 = -1$

i) 
 $x - 1 - 1 = 1$


Let us add 2 brown squares to the both sides of the equation


 $x - 1 - 1 + 1 + 1 = 1 + 1 + 1$


The 2 brown squares will cancel or neutralize the other 2 blue squares, leaving the answer



 $x = 1 + 1 + 1 \Rightarrow x = 3$

ii) $x + 2 = -1$


 $x + 1 + 1 = -1$

Let us add 2 blue squares to the both sides of the equation


 $x + 1 + 1 - 1 - 1 = -1 - 1 - 1$


 $x = -1 - 1 - 1 \Rightarrow x = -3$

Activity 4: Factorizing Quadratic Equations

The researchers ensured that students in the E and C groups possessed a clear comprehension of quadratic equations, which they defined as equations of the type $ax^2 + bx + c = 0$, where 'a,' 'b,' and 'c' represent coefficients and the task was to determine the values of 'x' that satisfy the equation. A quadratic equation, such as $x^2 + 5x + 6 = 0$, was given to each group in the E and C groups to factor using the appropriate algebra tiles.

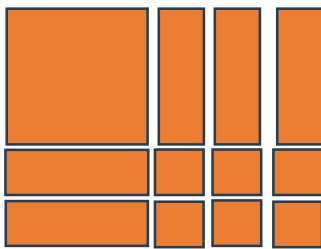
Step1: Modeling the quadratic equation with algebra tiles:



$$x^2 + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1 = 0$$

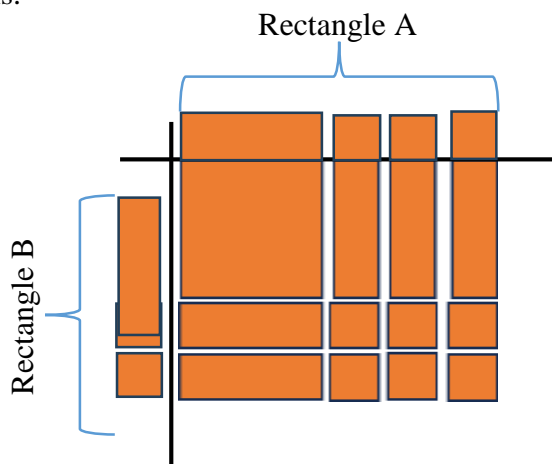
Step2: Arranging Tiles

Create a rectangular array of tiles by positioning big square tiles in the top-left corner and one-unit tiles in the bottom-right corner. Students arrange the tiles to form a rectangle. This may require some manipulation, such as sliding tiles around to make a perfect rectangle, as shown below.



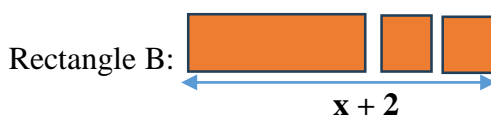
Step3: Factoring the Rectangle:

At this stage, the researchers clarified to the students that their task was to break down the rectangular shape formed into two smaller rectangles, which is the fundamental idea behind factoring quadratic equations.



Step4: Finding Factors:

Students were assigned the challenge of determining the dimensions of rectangles A and B, which make up the larger rectangle created earlier. These dimensions will be used as the factors of the quadratic equation $x^2 + 5x + 6 = 0$.



The students were instructed to grasp the concept that the expressions $(x + 2)$ and $(x + 3)$ represent the roots of the quadratic equation $x^2 + 5x + 6 = 0$, which aligns with the factored representation of the quadratic equation, $(x + 2)(x + 3) = 0$.

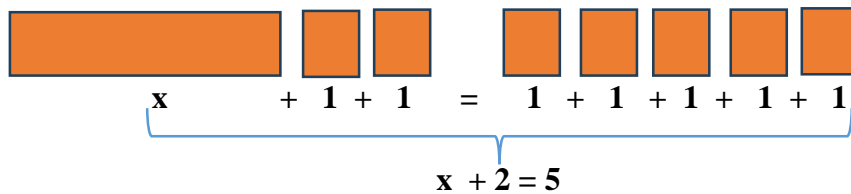
Step5: Solving for the Variable 'x':

The researchers demonstrated to the students that they possessed two binomial expressions that were both equal to zero. They had the option to equate each factor to zero and subsequently determine the value of x . In this instance, they would resolve equations like $x + 2 = 0$ and $x + 3 = 0$ to ascertain the values of x . Solve for x to find the roots or solutions of the quadratic equation. In this case, $x = -2$ and $x = -3$.

Activities 5: Translation of Word Problems

The goal of this task was to empower students in the E and C groups to convert word problems into linear equations and then use algebra tiles to solve them.

Ama attended school and used a portion of the Ghs 5 she received for lunchtime expenses to purchase sweets. Afterwards, she had only GHS 2 left. How much money did she use to buy the sweets? Let the word problem be represented as $x + 2 = 5$, the variable x represents the amount spent on the sweets.



Solve for x to find the roots or solutions of the linear equation. In this case, $x = 3$, meaning she spent GHS 3 on the sweet.

Week 5 Activities

Posttest

Following the intervention, both groups took a posttest again to determine the extent of their understanding of algebraic expressions. The posttest questions were the same questions used in the pretest to evaluate the effectiveness of each manipulative tool.

Data Analysis

The pretest and posttest scores of both groups will be compared using statistical analysis techniques such as mean and standard deviation. The aim is to determine whether any significant differences exist between the control and experimental groups in terms of understanding algebraic expressions.

Table.1: The Analysis on How Pretest Questions were Answered by Both

Questions	Number of Students	
	E Group	C Group
1. What is an algebraic expression?		
2. How do you identify the terms in an algebraic expression?		
3. Explain the difference between a coefficient and a constant in an algebraic expression.		
4. How can you simplify an algebraic expression by combining like terms?		
5. What does it mean to evaluate an algebraic expression?		
6. How do you determine the value of a specific variable in an algebraic expression?		
7. Explain the meaning of the distributive property in relation to algebraic expressions.		

8. Give an example of an algebraic expression that represents “twice a number increased by 5.”	12	25
9. How can you use an algebraic expression to solve real-life problems or mathematical equations?		
10. Describe the steps to simplify the expression $3(x + 2) - 5x$.	39	45

According to the data provided in Table 1, it is evident that none of the students in either the Experiment group (E) or the Control group (C) responded correctly to questions 1 to 7 and question 9. Concerning question 8, only 12 students in the E group (24%) answered it correctly, while the remaining 76% were unsuccessful. In the C group, 25 students (50%) were successful in answering question 8 while the other 50% were not. Furthermore, 39 students in the E group (78%) answered question 10 correctly, with the remaining 22% unable to do so. In the C group, 45 students (90%) answered question 10 correctly, while the remaining 10% could not.

Table.2: The Frequency Distribution of Pretest Marks

Marks Percentage	Frequency for E Group	Percentage %	Frequency for C Group	%
0	22	44	26	52
1	11	22	10	20
2	9	18	4	8
3	7	14	6	12
4	1	2	2	4
5	0	0	2	4
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
TOTAL	50	100	50	100

Source: Field Data

Based on the data provided in Table 2, the test scores for a total of 100 students (50 students in each group) were assessed on a scale of 0 to 10. The analysis reveals that out of the students, 48 individuals, equivalent to 96%, achieved a score of 0. Moreover, 21 students, accounting for 42%, attained a score of 1. Similarly, 13 students, constituting 26%, secured a score of 2 while another 13 students, also making up 26%, received a score of 3. Additionally, 6% of the students, which corresponds to 3 students, earned a score of 4, whereas 4% of the students, or 2 students, managed to obtain a score of 5. In terms of the average performance across both groups, the mean score is calculated to be 1.08. This implies that, on average, the students achieved a score of 1.08 out of a total of 10 marks.

The standard deviation serves as an indicator of how much variation or spread exists among the scores. A larger standard deviation indicates that the scores are more widely dispersed from the mean. The standard deviation for both the pretest groups is quite similar, specifically 1.2 for the Experimental group and 1.4 for the Control group.

In interpreting the standard deviations of 1.2 and 1.4 within this context, it signifies that the scores of the 100 students tend to diverge from the mean score of 1.08 by around 1.2 and 1.4 respectively. Stated differently, the majority of students' scores would fall within 1.2 points above or below the mean. The data in Table 2 clearly demonstrates that nearly all students from both groups achieved scores below the average of 5 marks out of 10 marks.

Hence, a total of 98 students, constituting 98% of both groups, obtained scores below 5. These results strongly indicate that the Basic seven (7) students enrolled in Bagabaga Demonstration Junior

High School are facing significant difficulties in comprehending the concept of Algebraic expressions. This underscores the necessity for an intervention to be implemented to help them surmount this obstacle.

Based on the results of the pretest, which indicate that students across both the Experiment (E) and Control (C) groups struggled with understanding algebraic expressions, it is clear that targeted intervention is necessary to improve their comprehension in this area.

Table.3: The Analysis on How Posttest Questions were Answered

Questions	Number of Students	
	E Group	E Group
1. What is an algebraic expression?	50	50
2. How do you identify the terms in an algebraic expression?	50	50
3. Explain the difference between a coefficient and a constant in an algebraic expression.	50	50
4. How can you simplify an algebraic expression by combining like terms?	42	35
5. What does it mean to evaluate an algebraic expression?	50	50
6. How do you determine the value of a specific variable in an algebraic expression?	50	30
7. Explain the meaning of the distributive property in relation to algebraic expressions.	50	38
8. Give an example of an algebraic expression that represents “twice a number increased by 5.”	50	25
9. How can you use an algebraic expression to solve real-life problems or mathematical equations?	46	36
10. Describe the steps to simplify the expression $3(x + 2) - 5x$.	50	45

Based on the information provided in Table.3, it is evident that all students in the Experimental group (E) and the Control group (C) were able to answer questions 1, 2, and 3, which signifies a 100% enhancement compared to their performance in the Pretest. As for question 4, notable progress is observed: 84% of the students in the Experimental group successfully addressed it, while the remaining 16% encountered difficulty.

This is a substantial advancement from their Pretest outcomes. In the Control group, 70% of the students answered it correctly, with only 30% unable to do so. Importantly, every student from both groups managed to respond accurately to question 4. All students in the Experimental group responded to questions 6, 7, 8, and 10, yielding a completion rate of 100%. In contrast, the Control group displayed varying rates of participation: 60% answered question 6, 76% tackled question 7, 50% addressed question 8, and 90% responded to question 10. Although these results showed that there was a tremendous improvement over the Pretest, the Experimental group performed better than the Control group.

Table.4: The Frequency Distribution of Posttest Marks

Marks	Frequency for E Group	Percentage %	Frequency for C Group	Percentage %
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

5	1	4	5	10
6	1	2	1	2
7	2	6	4	8
8	4	8	3	6
9	12	24	10	20
10	30	60	27	54
Total	50	100	50	100

Source: Field Data

According to the data provided in Table.4, the test scores for the 100 students, with 50 in each group, were graded on a scale of 10 marks. Notably, no student from either group received a score below the mean score of 5. Furthermore, in the Experimental group, 60% of the students, equivalent to 30 students, successfully answered all 10 questions, while in the Control group, 54% of the students, amounting to 27 students, also accomplished this feat.

DISCUSSION

This section analyzes the statistical data from the experimental and control groups, using a one-way ANOVA (Analysis of Variance). This test will help determine whether there are any statistically significant differences between the means of the two groups after the intervention.

- Null Hypothesis (H_0): There is no significant difference between the mean scores of the experimental and control groups.
- Alternative Hypothesis (H_1): There is a significant difference between the mean scores of the experimental and control groups.
- Experimental Group (hands-on manipulatives):
 - Mean = 7.3
 - Standard Deviation = 5.9
 - Sample Size = 50
- Control Group (virtual manipulatives):
 - Mean = 8.86
 - Standard Deviation = 1.5
 - Sample Size = 50

Python was used to determine the outcome.

The results of our ANOVA analysis are as follows:

- F-ratio = 3.283
- Critical F-value = 3.938 ($\alpha = 0.05$)

Since the calculated F-ratio (3.283) is less than the critical F-value (3.938), we fail to reject the null hypothesis. This means that there is no statistically significant difference between the mean scores of the experimental group (using hands-on manipulatives) and the control group (using virtual manipulatives) in understanding the concept of Algebraic Expressions.

RECOMMENDATIONS

Based on the ANOVA analysis, which found no statistically significant difference between the posttest scores of students using hands-on manipulatives versus those using virtual manipulatives to learn Algebraic Expressions, the following recommendations for fellow researchers, teachers, and educators are made.

For Researchers

1. Replicate the Study:
 - Conduct similar studies with larger sample sizes and diverse populations to confirm these findings.
 - Consider longitudinal studies to observe the long-term effects of both types of manipulatives.
2. Investigate Additional Variables:
 - Examine other variables that might influence learning outcomes, such as teaching methods, student engagement, or prior knowledge.
 - Explore whether the combination of hands-on and virtual manipulatives yields better results than using either method alone.
3. Qualitative Research:
 - Conduct qualitative research (e.g., interviews, focus groups) to gain insights into students' and teachers' experiences and preferences regarding both types of manipulatives.

For Teachers and Educators

1. Diverse Approaches:
 - Use a blend of hands-on and virtual manipulatives to cater to different learning styles and preferences among students.
 - Integrate manipulatives with other instructional strategies to enhance understanding and retention.
2. Student-Centered Learning:
 - Pay attention to individual student responses to different types of manipulatives. Some students might benefit more from one type than the other.
 - Provide opportunities for students to choose the type of manipulative they prefer, fostering a more personalized learning experience.
3. Professional Development:
 - Engage in ongoing professional development to learn effective ways to integrate both hands-on and virtual manipulatives into the curriculum.
 - Share best practices and experiences with colleagues to improve collective teaching strategies.

For Policy Makers and Educational Leaders:

1. Resource Allocation:
 - Ensure schools have access to both hands-on and virtual manipulatives, recognizing that different tools can be effective in different contexts.
 - Invest in teacher training programs that focus on the effective use of both types of manipulatives.
2. Curriculum Design:
 - Develop curricula that incorporate both hands-on and virtual manipulatives, providing guidelines on how to use them effectively.
 - Encourage the inclusion of manipulatives in assessment strategies to evaluate their impact on student learning.

CONCLUSION

The outcome suggests that both hands-on and virtual manipulatives can be effective in teaching Algebraic Expressions, but no significant difference was found between the two. It is essential to continue researching, experimenting, and sharing knowledge to optimize the use of these tools in educational settings. Emphasizing a balanced and flexible approach that considers individual student needs and preferences will likely yield the best educational outcomes.

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